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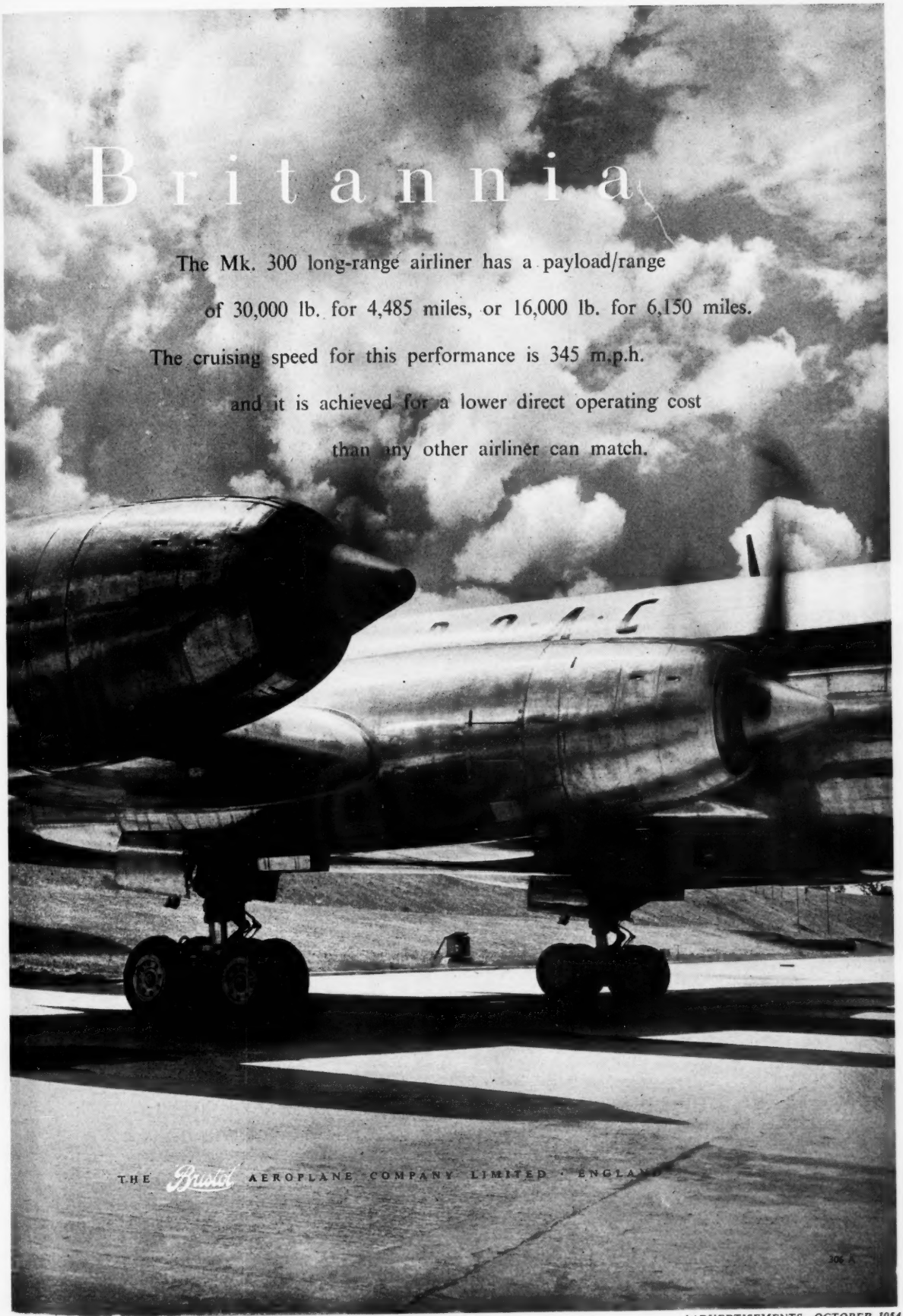
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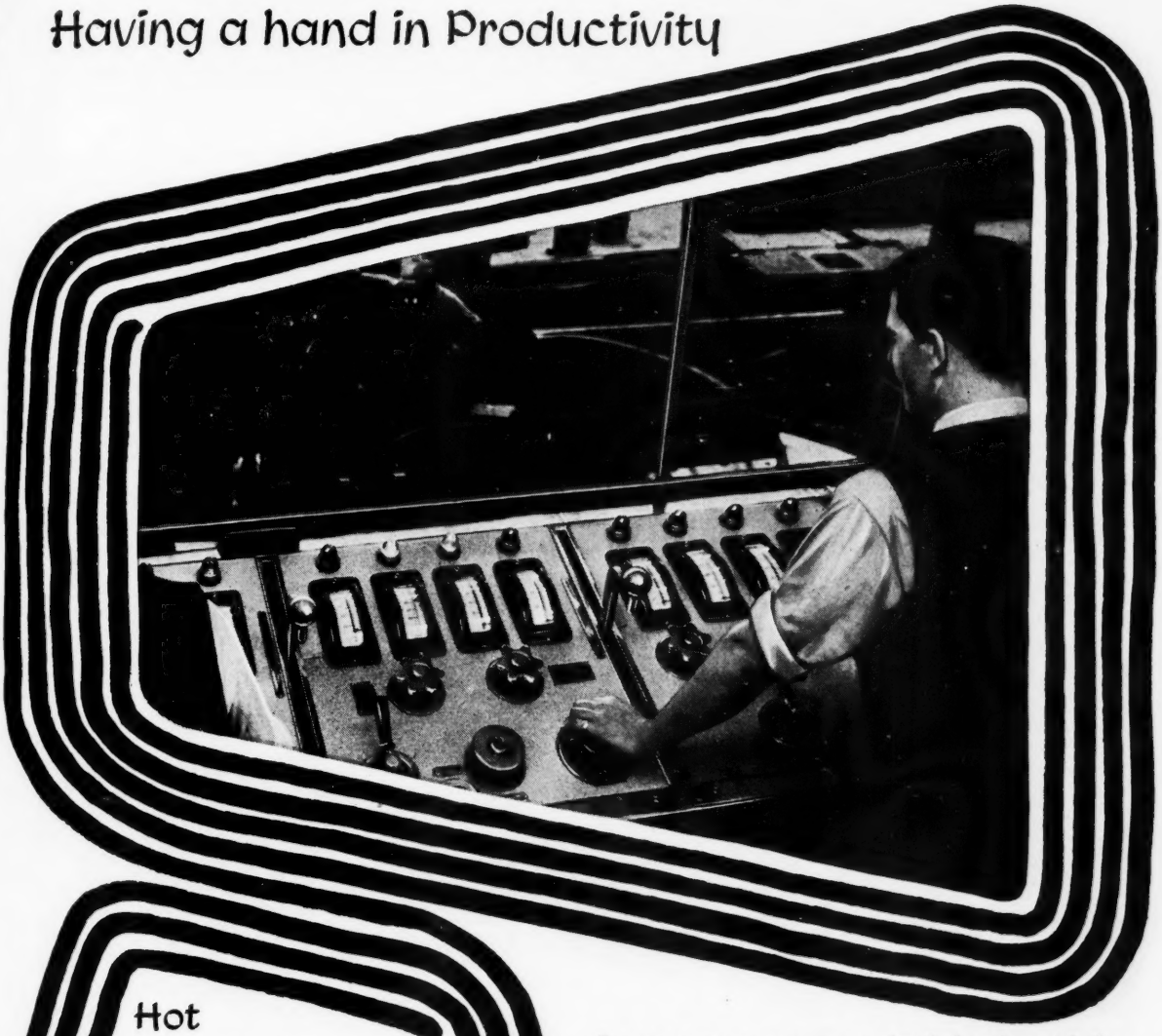

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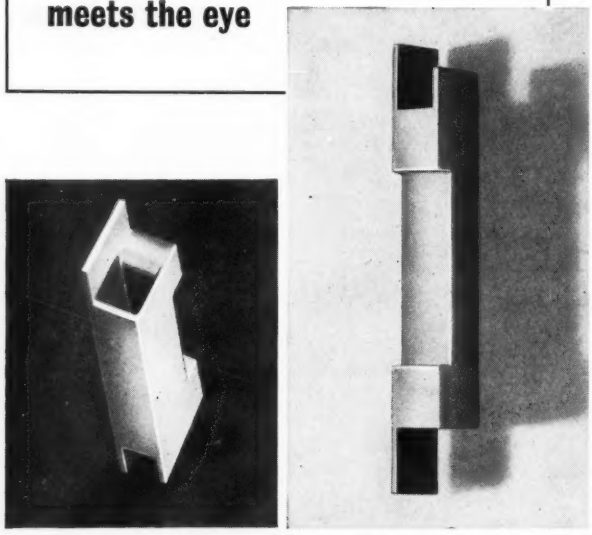

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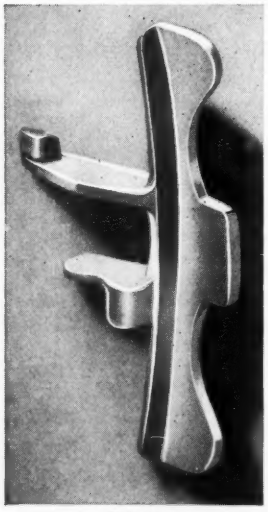

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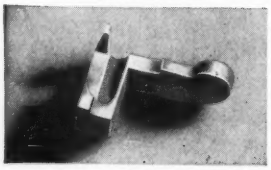

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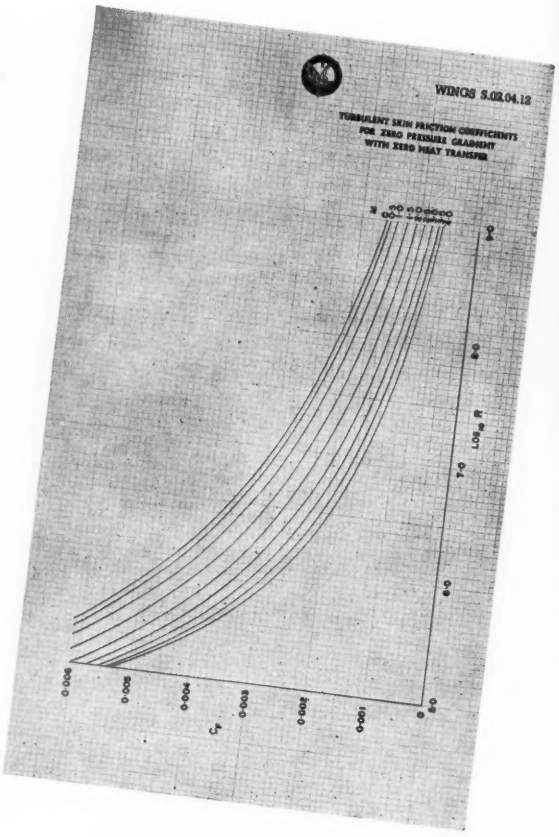

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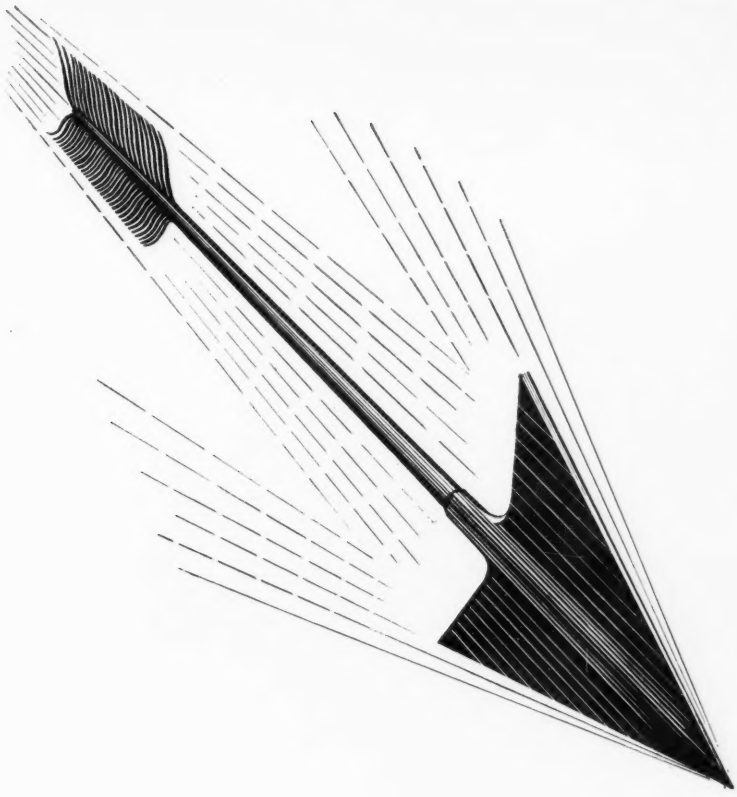

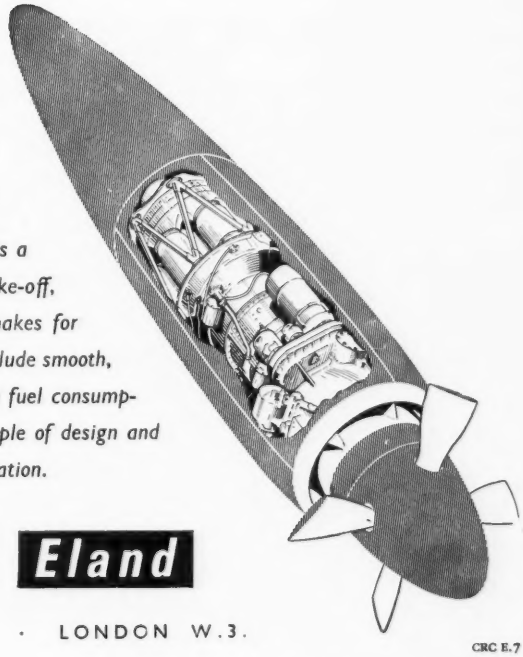

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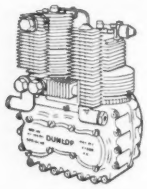
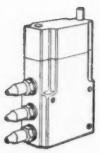
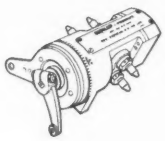
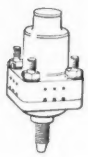
THE MARK I **COMPRESSOR**  *a 3,000 p.s.i. supercharged unit maintaining ground level performance up to 30,000 ft.*

 **SELECTOR VALVES** *of various types for use in conjunction with pneumatic rams to operate all aircraft services.*

The **INFINITELY VARIABLE**  *follow up control for multi-position flap operation,* **PRESSURE RELIEF VALVES**  *with a wide selection of blow-off pressures and supplementary units such as* **FUEL PRESSURISATION VALVES** *which deliver a high rate of flow at low output pressure.*

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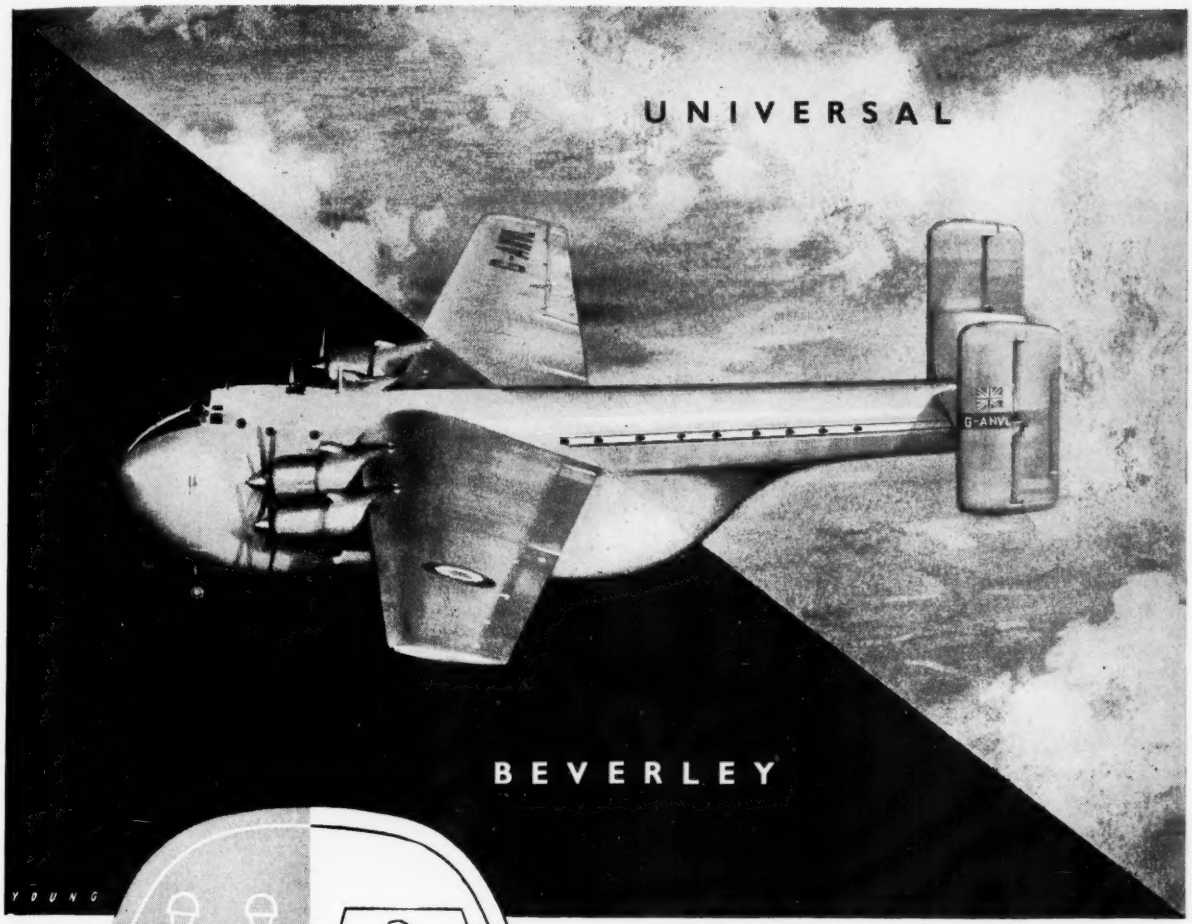

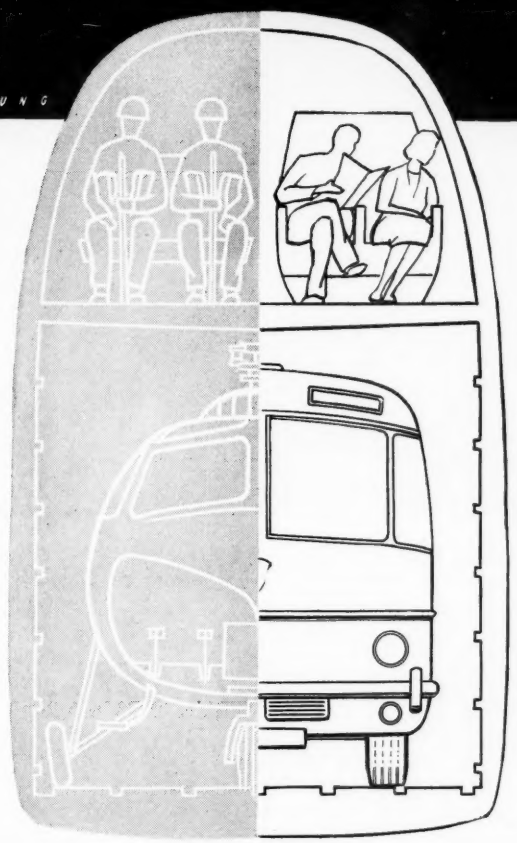

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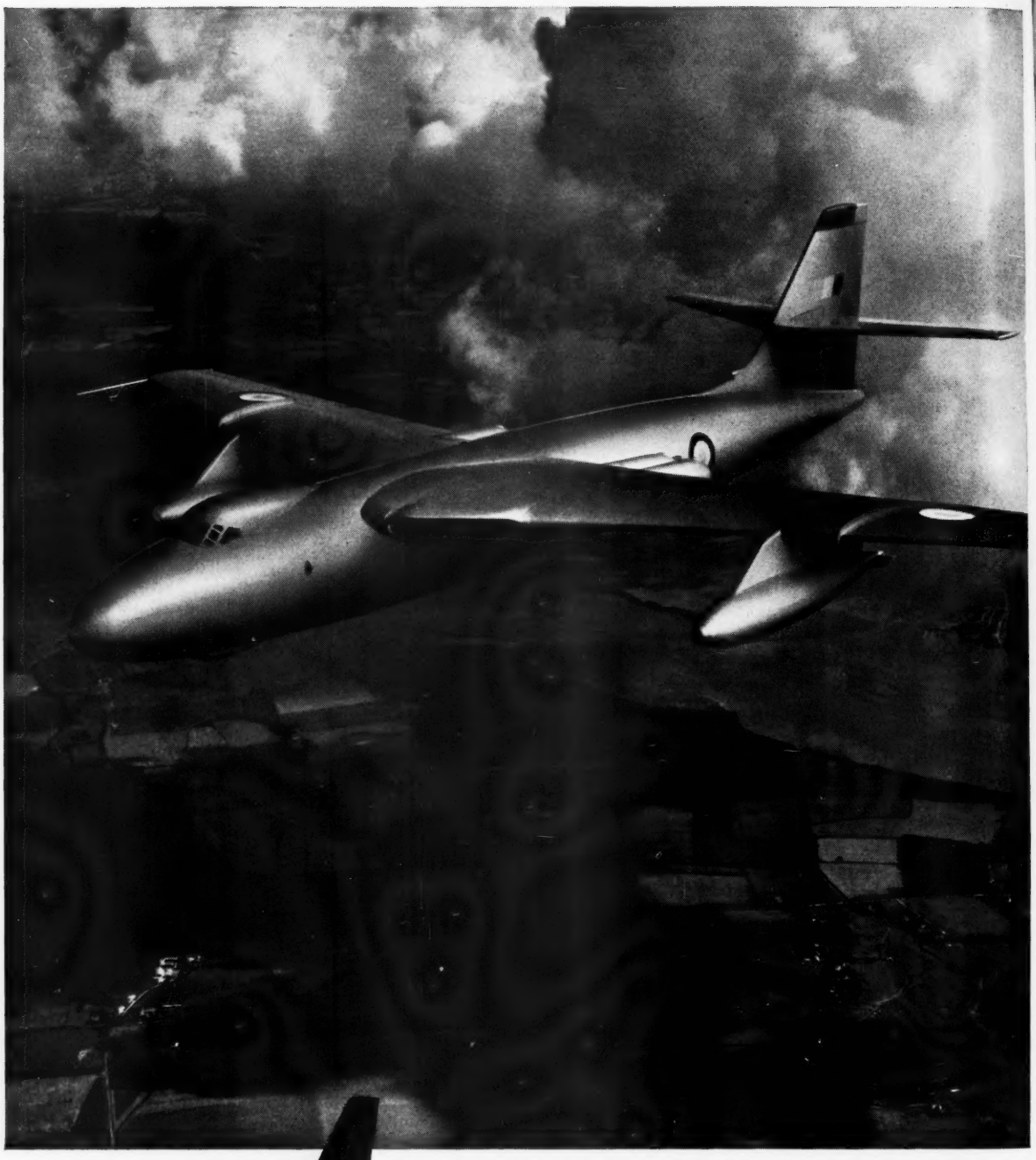

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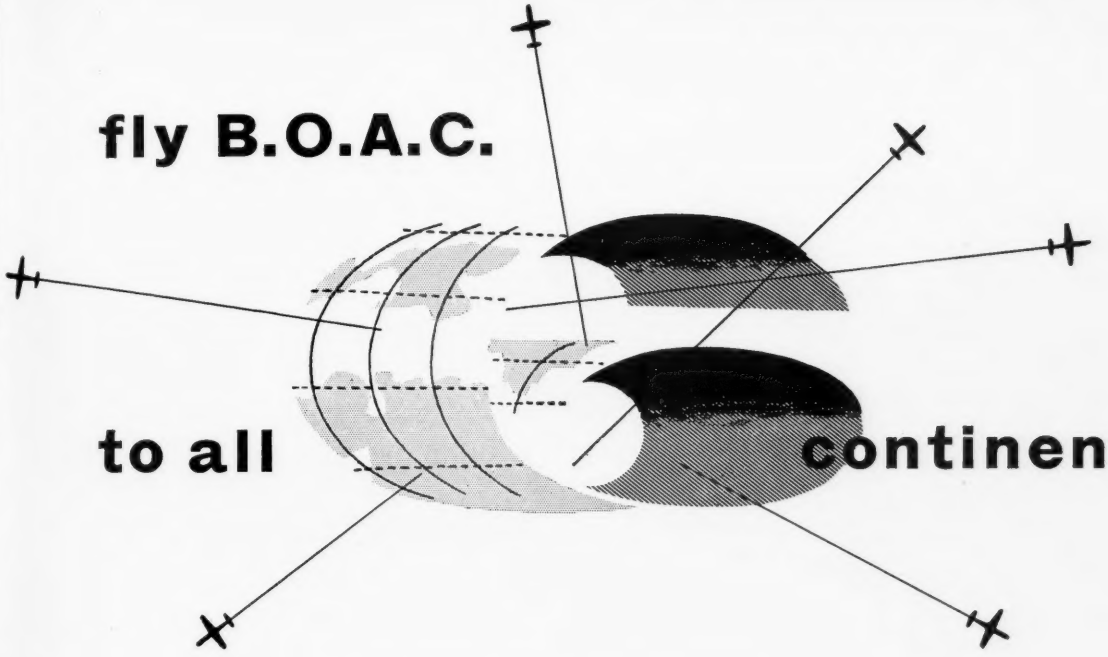

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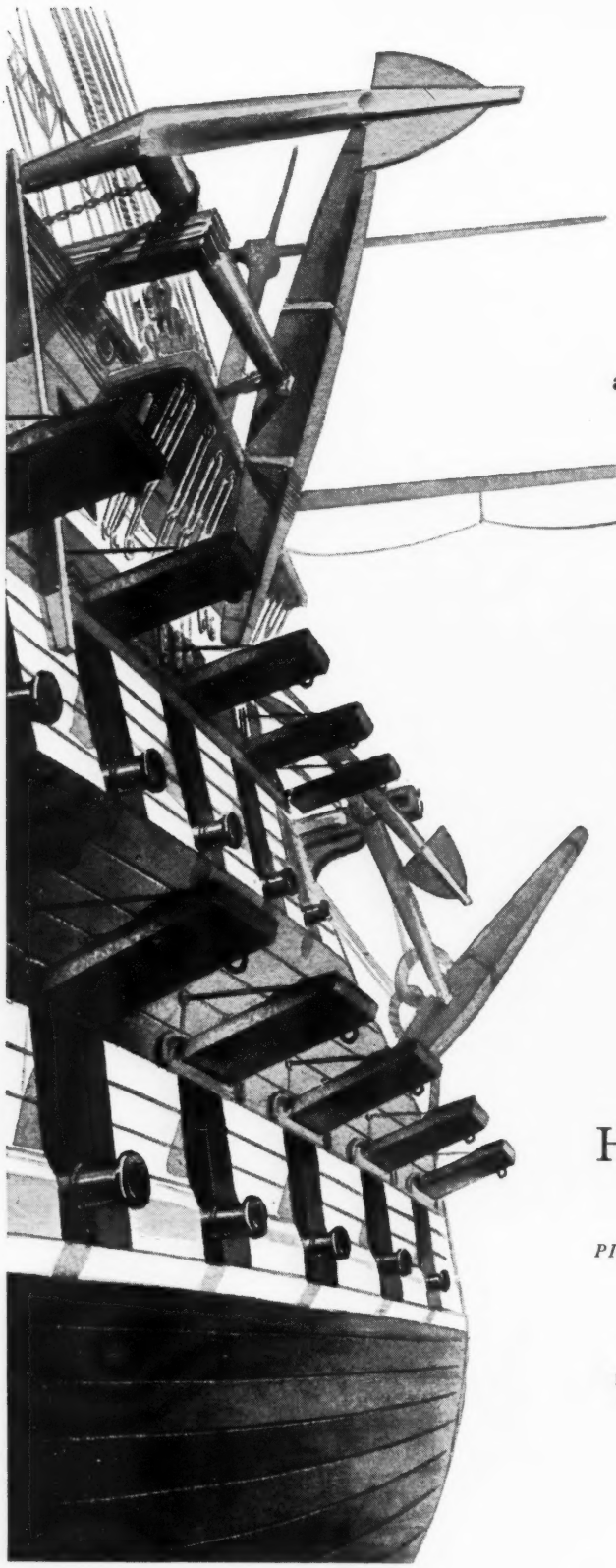

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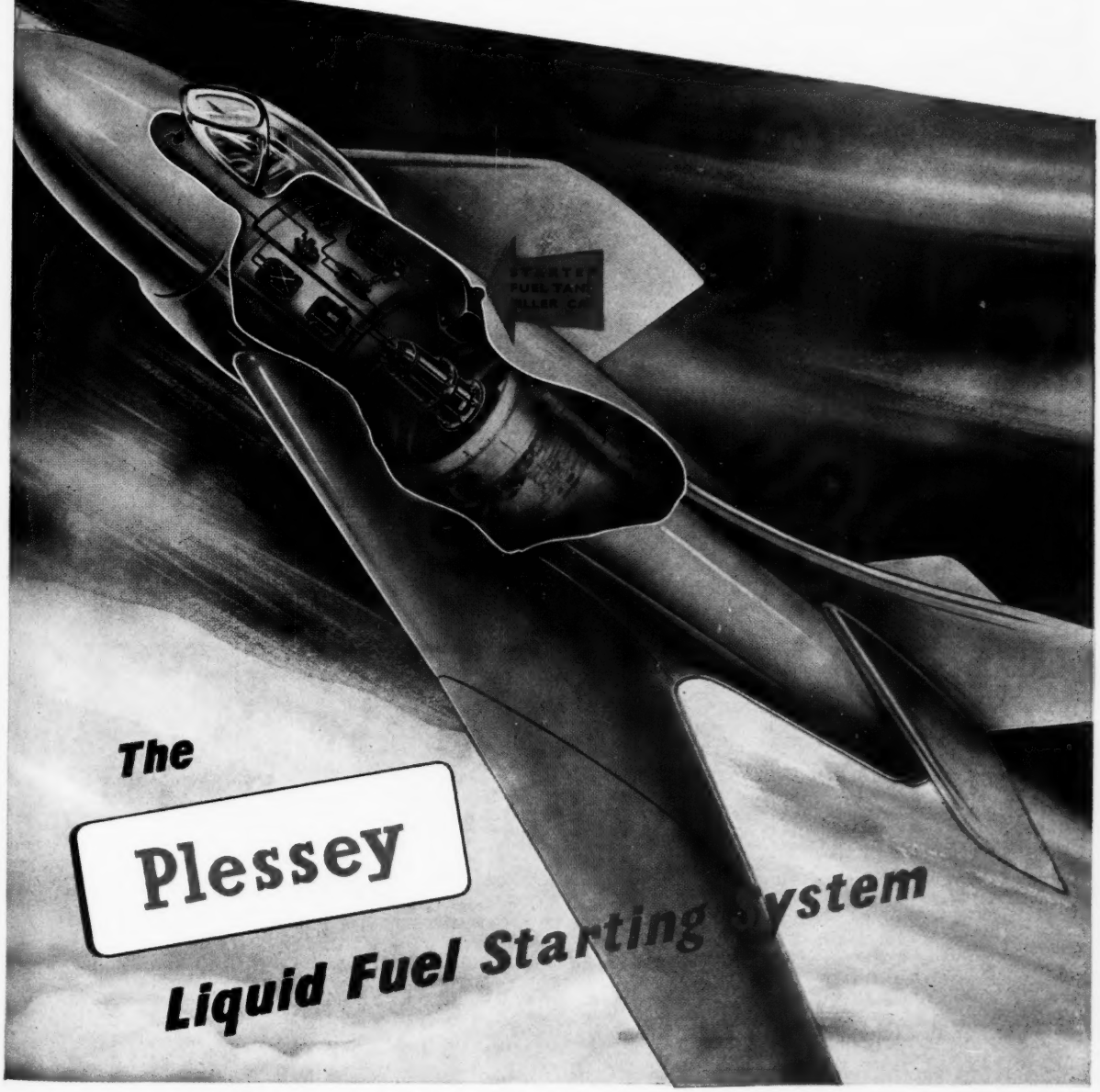

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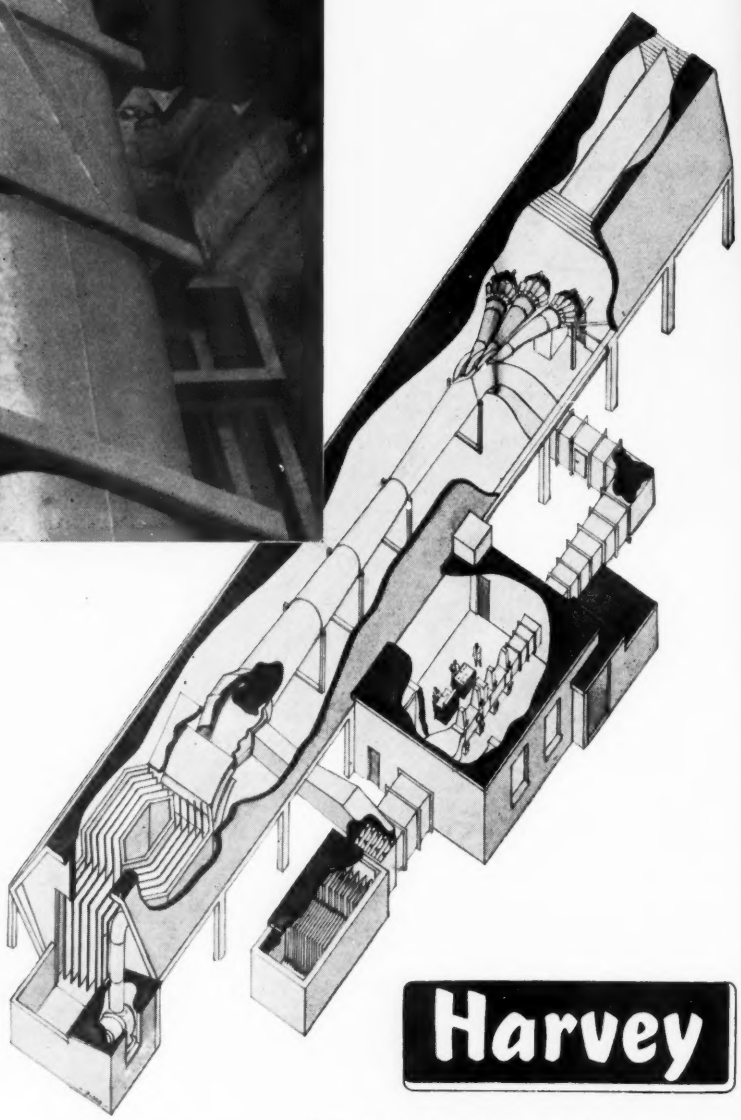

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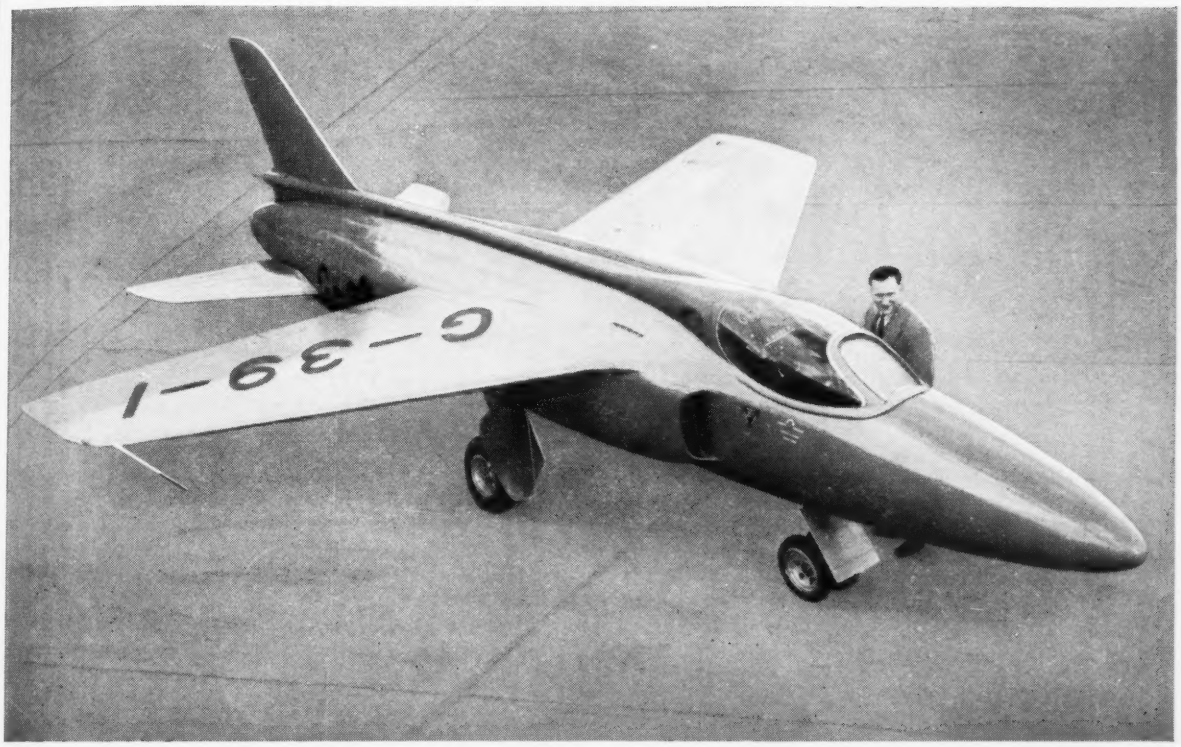

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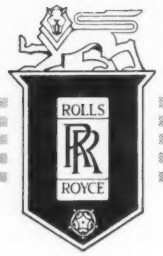

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VOLUME 58 OCTOBER 1954 NUMBER 526

CONTENTS

PAGE

Notices XLIX

Fifth Anglo-American Conference—Associate Fellowship Examination—Society of British Aircraft Constructors' Educational Grants—News of Members—Birthday Honours—Graduates' and Students' Section: Reception and Dance: And Visit—Diary—Elections—Changes of Address.

The Seventh Louis Blériot Lecture—
The Domain of the Helicopter — RAOUL HAFNER — 661

The Analysis and Synthesis of Vibrating Systems — R. E. D. BISHOP — 703

Technical Notes 720

The Level of Safety Achieved by Periodic Inspection for Fatigue Cracks—R. R. SHAW. Empirical Formulae for the Determination of Stress Curves—A. C. NICHOLLS. Definition of Specific Impulse—S. W. GREENWOOD, F. W. THORNE, K. ASQUITH and J. G. WHITAKER. The Flexural Centre or Centre of Shear—J. A. JACOBS.

The Library 729

REVIEWS:—Hydraulic Systems and Equipment—Electronics—Theory and Design of Steam and Gas Turbines—Die Eroberung Des Luftreiches—Proceedings of the Society for Experimental Stress Analysis, Vol. XI, No. 2—Characteristics and Applications of Resistance Strain Gauges—V2—The Amazing Mr. Doolittle—Materials and Processes—Textbook of Physics—Funfstellige Tafel Der Trigonometrischen Funktionen—Higher Transcendental Functions, Vols. I and II.

ADDITIONS TO THE LIBRARY. REPORTS.

All communications for publication in, and advertisements for, the JOURNAL *should be addressed to the Editor,* JOAN BRADBROOKE, *A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary,* A. M. BALLANTYNE, *T.D., B.Sc., Ph.D., A.F.I.A.S., A.F.R.Ae.S.*

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless stated otherwise.

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NOTICES

An annual sum of £250 is available for premium awards for papers, including Technical Notes, published in the Journal. Members and non-members of the Society are invited to submit papers on any aspect of aeronautics

Fifth Anglo-American Conference

The Fifth Anglo-American Aeronautical Conference will be held in Los Angeles, California, from 20th June to 1st July 1955. An application form for membership of the Conference, giving details of the travel and hotel arrangements, is enclosed in this JOURNAL.

News of Members

WING COMMANDER R. M. ALDWINCKLE (*Associate Fellow*) has been appointed R.C.A.F. Resident Engineering Officer to Canadair Ltd. in Montreal.

A. F. BOOT (*Graduate*) has taken up an engineering post with Canadair Ltd. in Montreal.

F. BUTTERS (*Companion*) has been appointed Deputy Public Relations Manager at F. C. Pritchard, Wood and Partners Ltd., London.

D. CAMPBELL-ALLEN (*Associate Fellow*) has taken up an appointment as Senior Lecturer in Civil Engineering at the University of Sydney.

P. E. FLEGATE (*Associate Fellow*) has been appointed Chief Development Engineer (Engine Controls) at the Cheltenham factory of Smiths Aircraft Instruments.

N. H. LE GALLAIS (*Associate Fellow*) has been appointed Test Engineering Supervisor, Gas Turbine Engineering Division, A. V. Roe (Canada) Ltd.

A. H. C. GREENWOOD (*Associate*) has been appointed Asistant General Manager (Service) of the Aircraft Division of Vickers-Armstrongs Ltd. and will be responsible for the after-sales service activities of the Weybridge and Supermarine works.

M. JACKSON (*Associate Fellow*) has now taken up a teaching appointment in the Engineering Department of the Burton-on-Trent Technical College.

A. W. LAUGHTON (*Associate*) has been appointed General Manager of Eaton Axles Ltd. of Great Sankey, Warrington.

N. S. MAKINS (*Associate Fellow*) is now employed as Development Engineer in the Sales Engineering Department of the R.T.S.C. Group.

B. A. PEASTER (*Associate Fellow*) is now employed as Assistant Designer, Projects, by Dowty Fuel Systems Ltd., Cheltenham.

K. T. SPENCER, C.B.E. (*Fellow*) has been appointed Chief Scientist, Ministry of Fuel and Power, in succession to Sir Harold Roxbee Cox (*Fellow*).

J. WINWARD (*Associate Fellow*) has recently been appointed Senior Test Equipment Engineer with the English Electric Co. Ltd., Whetstone.

S. C. WYBROW (*Associate Fellow*) has been appointed Principal Inspection Officer, Aeronautical Inspection Directorate, South Eastern Area, Ministry of Supply.

Graduates' and Students' Section—Reception and Dance

The Graduates' and Students' Section have arranged a Reception and Dance which will be held at the headquarters of the Society on Friday 29th October 1954 from 7.45-11.30 p.m. Tickets, price 7s. 6d. single and 13s. 6d. double (including refreshments), are obtainable from members of the Committee or by post from Mr. E. J. Catchpole, 12 Amery Road, Harrow-on-the-Hill, Middlesex. Members of the Section are requested to enclose a stamped addressed envelope with their remittance.

Birthday Honours

Wing Commander R. Milroy Hayes (*Associate*), recently returned from Pakistan, was awarded a C.B.E. in the Birthday Honours.

Associate Fellowship Examination

The next Associate Fellowship Examination will be held in the offices of the Society on 20th, 21st and 22nd December 1954. All candidates who have entered for the Examination will be informed of the arrangements.

Society of British Aircraft Constructors' Educational Grants

The following were awarded Educational Grants by the Society of British Aircraft Constructors at a Selection Committee meeting held in the offices of the Royal Aeronautical Society on 26th July 1954:—

M. D. CHAMBERLAIN, Spalding Road, Pinchbeck, Spalding, Lincs. (To serve his apprenticeship with The de Havilland Engine Co. Ltd.)

B. J. FINES, Wingland, Sutton Bridge, nr. Spalding, Lincs. (To serve his apprenticeship with The de Havilland Aircraft Co. Ltd.)

R. F. H. OWSTON, 20 Ashlea Drive, Giffnock, Glasgow. (To serve his apprenticeship with The de Havilland Aircraft Co. Ltd.)

A. E. PRYCE, 6 Bryn Avenue, Rhos-on-Sea, Colwyn Bay. (To serve his apprenticeship with The de Havilland Aircraft Co. Ltd.)

R. DE SAEGER, 269 Battle Road, Hollington, St. Leonards-on-Sea. (Subject to acceptance by Rolls-Royce Ltd.)

The Scholarship Selection Board consisted of representatives of the Society of British Aircraft Constructors and of the Royal Aeronautical Society.

Graduates' and Students' Section—Visit

A visit has been arranged to the Royal Aircraft Establishment for Wednesday 13th October 1954 at 10.30 a.m. Lunch will be available. Visitors will be able to see the wind tunnel, structural and engine test rigs and the flight test department.

The numbers for this visit are strictly limited and applications should be sent not later than 6th October to the Honorary Visits Secretary, P. D. Stewart, 217 High Road, East Finchley, N.2. Applicants are asked to state their grade of membership of the Society and that they are British subjects.

Elections

The following is a list of new members and transfers of membership of the Society:—

Associate Fellows

Terence Henry John Heffernan (*from Graduate*)
Thomas James Taylor (*from Graduate*)
Kenneth Ernest Yates (*from Graduate*)

Graduates

John Allcroft (*from Student*)
Bernard Ashworth Hodson
Addanki Venkata Rangarao

Students

Nikhil Chandra Bhattacharjee
Herman Hendrik Dalman
Alistair Warner McCormick
Nilkanth Janardan Shendge
William John Sinclair
Saran Kumar Srivastava

Companion

Frederick Eugene Anthony Jackson

Erratum

We regret that in the September JOURNAL there was a misprint in the entry for P. K. Digby, *Associate Fellow*, whose full name should read Paul Kichenside Digby.

Diary

LONDON

October 12th
SECTION LECTURE. Behaviour of Light Alloys at Elevated Temperatures. E. R. Gadd. 4 Hamilton Place, W.1. 7 p.m.

October 19th
GRADUATES' AND STUDENTS' SECTION. The Propeller-Turbine in Airline Service. T. M. Corson. 4 Hamilton Place, W.1. 7.30 p.m.

October 21st
MAIN LECTURE. The Development of Re-Heat. J. L. Edwards. At the Institution of Mechanical Engineers, 1 Birdcage Walk, S.W.1. 6 p.m. (Tea 5.30 p.m.)

October 29th
GRADUATES' AND STUDENTS' SECTION. Reception and Dance. 4 Hamilton Place, W.1. 7.45 p.m.

November 2nd
SECTION LECTURE. Design of Small Jet Engines. H. S. Rainbow. 4 Hamilton Place, W.1. 7 p.m.

November 4th
GRADUATES' AND STUDENTS' SECTION. Film: The German A4 Rocket (Operation Backfire). 4 Hamilton Place, W.1. 7.30 p.m.

November 11th
MAIN LECTURE. Boundary Layer Control. Dr. G. V. Lachmann. At the Institution of Mechanical Engineers, 1 Birdcage Walk, S.W.1. 6 p.m. (Tea 5.30 p.m.)

November 16th
SECTION LECTURE. Uses of the "Ace" Computer. Dr. E. T. Goodwin. 4 Hamilton Place, W.1. 7 p.m.

November 17th
GRADUATES' AND STUDENTS' LECTURE. Bird Flight and the Aeroplane. Captain J. L. Pritchard, C.B.E. 4 Hamilton Place, W.1. 7.30 p.m.

November 25th
SECTION LECTURE. The Development and Investigation of Fatigue Cracks. Dr. N. Thompson. 4 Hamilton Place, W.1. 7 p.m.

December 2nd
GRADUATES' AND STUDENTS' SECTION. Seaplanes—A Survey of the Development of Water-Based Aircraft Design. R. F. R. Storey. 4 Hamilton Place, W.1. 7.30 p.m.

December 9th
SECTION LECTURE. Problems of Structural Design. D. James. 4 Hamilton Place, W.1. 7 p.m.

December 16th
TENTH BRITISH COMMONWEALTH AND EMPIRE LECTURE. At the Assembly Hall of Church House, Westminster, at 6 p.m.

BRANCHES

October 7th
Brough. Aircraft Production Problems. L. R. Benton. Bristol Aeroplane Co. Ltd. Lecture Hall, Electricity Showrooms, Hull. 7.30 p.m.

October 8th
Birmingham. Power Units and the Economics of Civil Aviation. K. Nierop. Birmingham Chamber of Commerce. 7.30 p.m.

October 11th
Halton. Pilot Training for the R.A.F. Technical Branch. Squadron Leader H. G. A. Scilley. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

Henlow. Aircraft Engineering Operation in the Arctic. Major J. H. Webb. N. Beds. College of Further Education, Cauldwell Street, Bedford. 7.30 p.m.

October 12th
Belfast. Recent Developments in Production Processes. W. Goff. Kerr Room, Kensington Hotel, Belfast. 7 p.m.

October 14th
Isle of Wight. Some of the Problems of Bomber Design. D. Keith-Lucas, Saunders-Roe Club House, Church Path, E. Cowes. 6.30 p.m.

October 16th
Coventry. Visit to Pottery of J. Wedgwood & Sons, Barlaston, in the morning, and Trentham Gardens in the afternoon.

October 20th
Coventry. From Subsonic to Supersonic. Dr. Holder. The Wine Lodge, Coventry. 7.30 p.m.

October 25th
Halton. Helicopters. Sergeant Apprentice J. Harrod. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.
Henlow. The Case for the Night Fighter. P. A. Norman. Building No. 62, R.A.F. Technical College, Henlow. 7.30 p.m.

October 26th
Boscombe Down. Space Flight. John Humphries. Boscombe Down. 5.30 p.m.

October 27th
Halton. Visit to the College of Aeronautics, Cranfield.

November 2nd
Glasgow. Transonic Flight. M. J. Lithgow. (Subject to Confirmation.) Royal Technical College. 7.30 p.m.

November 4th
Brough. Main Society Lectures. Sir George Cayley Memorial Lecture. Captain J. L. Pritchard, C.B.E. Royal Station Hotel, Hull. 7 p.m.
Isle of Wight. Annual Dinner, Ryde Castle Hotel, Ryde.

November 8th
Henlow. The Schneider Trophy Races and Their Influence on Aircraft Design. Group Captain L. S. Snaith, C.B., C.B.E. Building No. 62, R.A.F. Technical College, Henlow. 7.30 p.m.

November 9th
Belfast. New Materials for Aircraft Construction. G. O. Jones. Kerr Room, Kensington Hotel, Belfast. 7 p.m.

November 18th
Isle of Wight. Junior Members' Evening.

November 19th
Belfast. Annual Dance. Malone House, Barnett's Park, Belfast. 9 p.m. till 2 a.m.
Brough. Annual Dinner Dance. Duke of Cumberland Hotel, North Ferriby.

November 22nd
Henlow. The Aeroplane and its Engine. Air Commodore F. R. Banks, C.B., O.B.E. The College of Aeronautics, Cranfield. 7.30 p.m.

December 1st
Glasgow. Any Questions. A social evening. St. Enoch Hotel. 7.30 p.m.

December 6th
Henlow. Film Evening. Building No. 62, R.A.F. Technical College, Henlow. 7.30 p.m.

December 8th
Brough. Fatigue. The Metallic Death-Watch Beetle. D. C. Smith and E. W. C. Wilkins. Lecture Hall, Electricity Showrooms, Hull. 7.30 p.m

December 9th
Isle of Wight. Annual General Meeting and film.

CHANGES OF ADDRESS

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters). New address (in block letters).
Grade of Membership. Old address

Changes of address must be received *before the 15th of the month* in order to be effective for the JOURNAL for the following month.

The Journal of the Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

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The Seventh Louis Bleriot Lecture

The Domain of the Helicopter

by

RAOUL HAFNER, F.R.Ae.S.

THE SEVENTH LOUIS BLERIOT LECTURE was given in Paris on 10th March 1954, by Mr. Raoul Hafner, F.R.Ae.S., under the auspices of the Association Française des Ingenieurs et Téchniciens de L'Aéronautique (A.F.I.T.A.)

The Lecture was attended by the President, Sir William S. Farren, C.B., M.B.E., F.R.S., F.R.Ae.S., and several members of Council of the Royal Aeronautical Society, by the Secretary, Dr. A. M. Ballantyne, T.D., A.F.R.Ae.S., members of the Society and of the British Aircraft Industry, and by a large and distinguished French audience.

Monsieur Jules Jarry, President of A.F.I.T.A., presided at the meeting and welcomed the guests before introducing Sir William Farren, President of the Society.

SIR WILLIAM, speaking in French, thanked Monsieur Jarry for his generous welcome and said that by his historic flight in 1909 Louis Blériot had joined their two countries and each year their Societies united to pay tribute to the memory of that famous man.

Introducing the Lecturer, Sir William said that the subject of Mr. Hafner's lecture, "The Domain of the Helicopter," was one on which his achievement entitled him to speak with authority, for Mr. Hafner was Chief Designer (Helicopters) of the Bristol Aeroplane Company and they were familiar with the helicopters which his Company had produced from his designs during the past few years.

Sir William Farren, President of the Society, on his arrival in Paris for the Seventh Louis Blériot Lecture, with Monsieur J. Jarry, President of A.F.I.T.A.

Most of them would welcome an aircraft which would bring them from Hyde Park in London to the Champs Elysées in Paris in, say, two hours. He thought they would be content if the journey took even a little longer, so long as it was regular, punctual and safe. He believed that Mr. Hafner was one of many who believed that the helicopter could, and soon would, give them such a service—completing as it were the work which Louis Blériot began 45 years ago. He had much pleasure in presenting to them their lecturer, Mr. Raoul Hafner.

Mr. Hafner then read in French, the introduction to his paper before handing over to Monsieur Morain, head of the Helicopter Department of S.N.C.A.S.O. and President of the "Commission de Giraviation" of A.F.I.T.A., who read an excellent French translation of the paper.

At the end of the paper brief speeches were made by Monsieur Louis Bréguet and Ingénieur en Chef Garry.

MONSIEUR BREGUET, the aircraft designer and pioneer of helicopter development in France, congratulated Mr. Hafner on the paper and thought Mr. Hafner's suggestion for a convertible helicopter was the most probable solution of the problem, although passengers' reactions to the gimbal-mounted seats would be a matter of much speculation. Monsieur Bréguet indicated that he had an alternative proposal for a propulsion system, within a similar design specification, but he agreed that as terminal delays could probably never be completely eliminated, the reduction of the actual travelling time between two points was the way to speed up communications and the convertible helicopter was the ideal solution.

INGENIEUR EN CHEF GARRY, head of the rotary-wing department of the French Air Ministry and holder of the first French helicopter pilot's certificate, referred to Mr. Hafner's work on rotorcraft design over the past 20 years and the interest of his department in Mr. Hafner's paper. He also mentioned B.E.A. plans for helicopters on internal air routes in Great Britain.

Following the lecture a reception was held and almost immediately afterwards, a Dinner was given at the French Aero Club which was attended by Madame Louis Blériot, Monsieur Louis Christiaens, French Secretary of State for Air, the British visitors and a number of leading French designers and technicians.

After the Dinner speeches were made by Monsieur Jarry, Monsieur Heurteux, President of l'Union Syndicate des Industries Aéronautiques, Sir William Farren and Monsieur Christiaens.

MONSIEUR JARRY, in a speech which covered many aspects of aeronautics, referred to the prospects of considerable development for the helicopter, or "gyro-aviation," which added a new burden to their already heavy one, of urgent aeronautical inventions. Mr. Hafner had dealt mainly with the civil domain of the helicopter but because of its mobility and freedom from elaborate runways and aerodromes he suggested that its military domain might be of even greater importance. He mentioned that there were several possible solutions for rapid take-off and landing from small runways and that for some time A.F.I.T.A. had been making investigations along those lines; recent realisations had justified their hopes in a remarkable way.

In conclusion Monsieur Jarry spoke of Anglo-French collaboration and the need for still greater co-operation and liaison: he gave a toast to the illustrious events of Anglo-French friendship with the hope, as expressed by Jean Cocteau when he wrote:

"*Dès que l'on parle de langage de l'amitié et du souvenir, les perspectives se reforment et l'immédiat devient éternel.*"

MONSIEUR HEURTEUX also spoke of the importance of the helicopter in a military role for the defence of national territory, and stressed the need for increasingly active collaboration between Great Britain and France in aeronautics.

SIR WILLIAM FARREN, again speaking in French, on behalf of the Society thanked the President and Council of A.F.I.T.A. for their hospitality and said that they regarded the Louis Blériot Lecture as second only in the great events of their year to the Wilbur Wright Lecture. They looked forward to welcoming next year in England a Frenchman who would deliver the next lecture to celebrate Louis Blériot's great flight.

He thought it was of the greatest importance that these outstanding lectures should be given by men whose own personal achievements entitled them to the highest respect. He had a preference for men at the height of their powers who could tell them not only of what they had done, but of what they were doing and expected to do.

Their lecturer that evening was a young man. He was trying to do something which they all applauded—to rationalise flying by bringing it into line with all other sensible means of transport. If he succeeded they should be able to stop in safety. His might not be the only way of achieving that end. It had to surmount formidable engineering difficulties. But the end itself was of the highest significance. They wished him the best of fortune.

Many of them seemed to have lost sight of the objective to which Mr. Hafner was devoting his life. Most of them were trying to contrive aircraft whose speed was so high that no one would be able to hear that they were coming. But unfortunately they left behind them a noise even louder and more uncomfortable than those to which they had become resigned, even if they did not enjoy them. Such quiet as remained in a somewhat uneasy world was being sadly disturbed by the machines of which they were so rightly proud. The aircraft was the only vehicle which resembled the citizen of the modern world in its inability to relax or to be quiet—as though he had infected it with his own uneasiness.

But he did not believe that they should fear that they were creating monsters which they could not control, or which must inevitably destroy the graciousness of life. It was not their machines which were to be feared, nor even, as some seemed to believe, a natural evil in men's minds. The greatest enemy of the peace they sought was fear itself. There was no security or safety to be found. It was a great Frenchman who said:

"*Tout est dangeroux ici-bas, et tout est nécessaire.*"

There was no assurance save in their own courage. If they ever felt any doubts, they need look no further than to those two struggles in which France was the world's battlefield.

It was in the air however, that the work they had to do must be done. Their Societies were the expression of their determination to do it, since only with knowledge could they hope to succeed. And it was meetings such as this which strengthened them, since from them they learned to understand one another.

MONSIEUR LOUIS CHRISTIAENS, Secretary of State for Air, described Anglo-French aeronautical co-operation as having begun with the thin thread of Louis Blériot's original crossing of the Channel, from which was constructed an aerial bridge, very strong and one of the most frequented in the world.

He emphasised the collaboration forged so strongly during the war and continuing still, and referred particularly to French production and development of British aircraft and engines. Monsieur Jarry had apologised for repeating himself and saying much of what had been said before, but could they grow weary of celebrating their friendship? Developing their co-operation, strengthening their bonds, they would be able to apply to themselves the words of Saint-Exupéry:

"*L'amitié ne consiste pas à se contempler face à face mais à marcher ensemble vers le même but.*"

On the following day most of the British guests visited the aerodrome at Issy - les - Moulineaux, the future "heliport" of Paris, to see two new French prototype helicopters, the S.N.C.A.S.O. Djinn and the S.N.C.A.S.E. Alouette, as well as the flight simulator helicopter Dorand.

SEVENTH LOUIS BLERIOT MEMORIAL LECTURE

The Domain of the Helicopter

by

RAOUL HAFNER, F.R.Ae.S.

Preface

When accepting the invitation to present the Seventh Louis Blériot Memorial lecture, the author was deeply conscious not only of the honour which was bestowed upon him by this invitation but, in an equal measure, of the responsibility which attached to it.

For the first time one of the eminent memorial lectures is devoted to the rotating wing; the helicopter has been raised to the level of discussion of the great aeronautical papers.

The natural impulse of the gentler sex when faced with a great occasion is invariably: "how shall one dress?" To the author, a somewhat similar problem became manifest: how can the helicopter, this newcomer, best be "dressed?" How should it be presented, so as to make not only a good impression but also a full and, above all, a true impression?

A complete treatise of the subject would embrace many volumes but only a small fraction of this vast scope can possibly be offered in a paper. Thus for instance, at one extreme, a panoramic view of the rotating wing could be presented which would necessarily lack information on details, whereas at the other extreme the investigation could be focused on one or two selected features which are typical of the helicopter and which could be dealt with exhaustively.

The author decided on following a middle course: In the first instance there is given an appreciation of the historical development of the helicopter (at least in general terms), and this is seen against the background of aeronautical activities as a whole, and culminates in a statement of achievements up to the present time.

As the subject as yet is new, a definition of the term helicopter has been given. On the technical side the fundamentals of the rotating wing are explained, and its performance and limitations established. Basic design parameters such as disc-loading, blade-loading and rotor weight are discussed, together with methods of rotor control, power supply, transmission fundamentals, and the basic differences between various rotor configurations. This leads on to a survey of the basic types of helicopter, their advantages and short-comings.

The paper could not be considered complete without reference to economics. This has been given in the form of a comparison, within the civil air transport field, of eight widely differing aircraft of which five are helicopters of various forms. Finally, it was felt that a reference to the diverse uses of the helicopter, besides that in the general air transport field, was needed.

An exposition such as has been given in this paper presents a very limited but—and this is the author's humble but earnest hope—an essentially balanced picture of the domain of the helicopter.

1. Introductory and Historic Remarks

1.1. GENERAL

Blériot, Butterfly, Comet, Honeybird, Sycamore, Albatross, Hornet, X-3—this assembly of words, selected at random, offers but a minute picture of the manifestations of flight, which are as numerous and diverse almost as those of life itself. Indeed, the types of wing that have cut through air is legion—small, large, slender and stubby wings; single, multiple, rigid, flexible and articulated wings; open or concealed wings—but the number of the basic types of wing is three:—

(*i*) the flapping wing
(*ii*) the fixed wing
(*iii*) the rotating wing.

The flapping wing is nature's great solution to this complex problem of dynamic flight. Although the physiological limitations of body control by means of muscle and nerve, which permit only of oscillatory movement in joints, have considerably diminished the scope for natural flight, nature's solution is overwhelming in its success. From the water take-off of the swan to the ceiling landing of the housefly stretches the gamut of nature's masterpieces, the ingenuity and complexity of which are quite beyond our powers of comprehension.

1.2. EARLY HISTORY

Man's first efforts to fly were endeavours to imitate the natural examples he saw around him. From the legendary flight of Daedalus to the early part of this century extends the cavalcade of man's persistent hope to take the air on flapping wings—but he failed. Indeed so complete was his failure that he did not even learn to appreciate how far away he still was from his objective.

A feeble imitator though he showed himself to be in the art and technique of nature, *homo sapiens* nevertheless made a masterstroke himself in the conception of the wheel. (Fig. 1). The technical evolution of man's civilisation began with the first use of the wheel, and this civilisation may be said to have run on wheels ever since. As time passed, the tasks for the wheel became more numerous and manifold, and it is significant that the great inventive genius Leonardo da Vinci, soon after

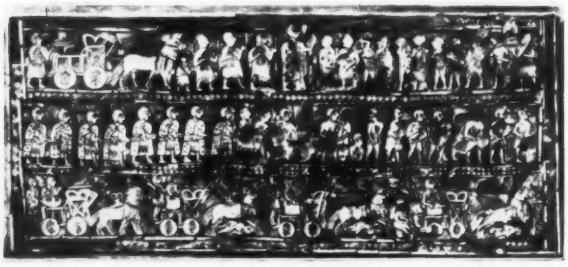


FIGURE 1. "The Standard of Ur."

his discovery of the screw-cutting lathe, had a vision of an altogether new application of the wheel[1, 2]. He thought of an aerial screw machine of 96 ft. diameter rotating about a vertical axis, built of iron and bamboo work and covered with linen cloth, and of which he said "I say that if this instrument made with a helix, is well made, that is to say of flaxen linen, of which one has closed the pores with starch, and is turned with great speed, the said helix is able to make a screw in the air, and to climb high . . ." A sketch of this machine drawn by his hand exists, and this document is generally regarded as the birth certificate of the rotating wing (Fig. 2). Da Vinci's mind was centuries ahead of his time and his idea remained undeveloped until the second half of the 18th century when, among others, M. Paucton and Sir George Cayley appear to have experimented with helicopter models[2].

It soon became evident that the rotating wing, being an easier engineering task than the flapping one, offered a better prospect of success (Fig. 3) and during the 19th century work on the latter was gradually retrenched in favour of the rotating wing. But this also failed to bring about the eagerly desired result of human flight. The reason for this failure—apart from the want of a light source of power—was the failure to appreciate that the real problem of dynamic flight lay not in obtaining sufficient lift, but in achieving stability and control.

A masterly performance is characterised by the complete disguise of its inherent difficulties, and nature has achieved this in truly full measure. The process of natural evolution has, it seems, brought about the improvement of the species by refinement, because the most developed and refined examples appear best to have withstood the hard test for survival. Thus, for instance, the combination of stability and elementary control, which is based on relatively simple elements, while it offered a fair measure of safety, had only a limited scope. Instability coupled with delicate control, on the other hand, appears to have offered a wider range of avenues for development which, in turn, progressively increased delicacy of control and the capacity for more complicated manoeuvres, until the advanced stage of natural flight, as we observe it now, was reached. Evidently the early investigators were completely misled by the elegance and ease in nature's methods of flight control; they were interested only in crude imitations of form which compared with the archetype like the dummy with the dancer. Gradually, however, the fullness of the problem revealed itself and the lesson was

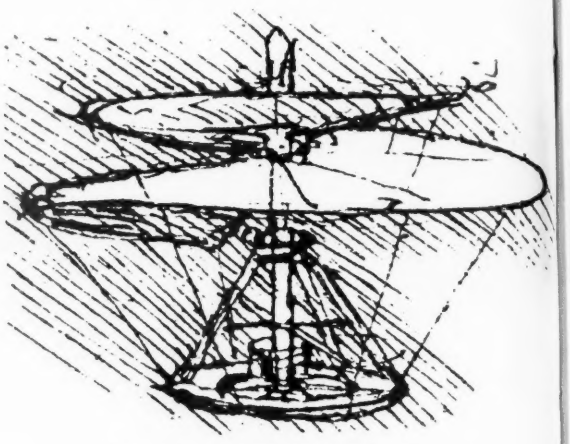


FIGURE 2. Leonardo da Vinci's helicopter.

learned. For the optimist, the goal which before had seemed almost within reach retreated into a dim distance ahead, and for the pessimist it had withdrawn altogether behind a seemingly unconquerable barrier. It was probably in such a mood of pessimism that Lord Kelvin, that great scientist and man of vision, rejected an invitation to become a Founder Member of the then Aeronautical Society with the following words[3] "I have not the smallest molecule of faith in aerial navigation other than ballooning, or of expectation of good results from any of the trials we hear of. So you will understand that I would not care to be a member of the Aeronautical Society."

There is, however, one form of flight which is evident through its inherent simplicity, a feature which did not escape the notice of some of the early investigators. It is the motionless glide—the flight of the kite. It seemed the form of flight most fitting for inanimate objects. If

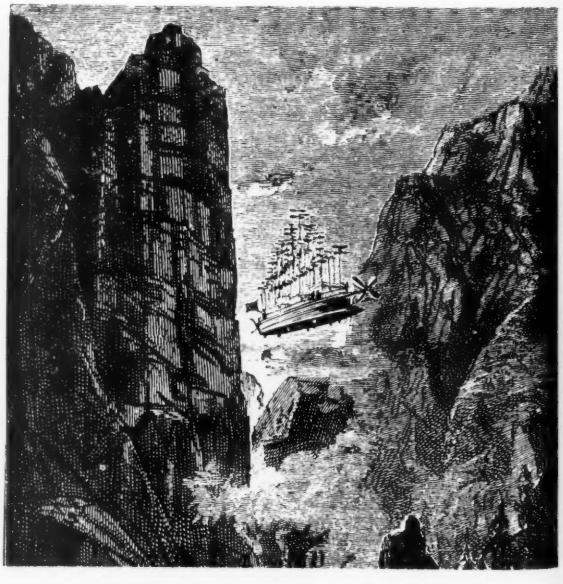


FIGURE 3. Jules Verne's helicopter.

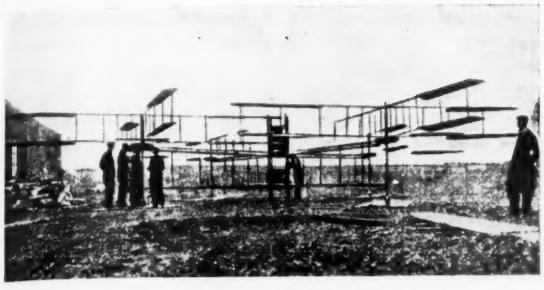


FIGURE 4. Bréguet-Richet helicopter (1907).

the other forms proved too difficult for man's mastery, this form perhaps was not beyond the possibility of achievement. Sir George Cayley probably came to this conclusion when he began his explorations with model gliders[2], and the thought gained ground quickly throughout the 19th century. Thus we find at the turning of this century a considerable number of people interested in what became known as the aeroplane and a few years later, in December 1903, this aeroplane took the air and controlled dynamic flight was at last achieved.

1.3. DEVELOPMENTS SINCE 1903

The relative simplicity of fixed-wing flight brought practical advances in rapid succession and the aeroplane stepped from victory to triumph. Flying meetings in open fields became the fashion and a cheering populace applauded the daring exploits of the pilots and the performance of their fixed-wing machines. The other wings, alas, were cast away and forgotten. But there remained a small minority who could not see in the spectacle of the triumphant fixed wing the alpha and omega of flying. They felt that human flight had been achieved at a considerable sacrifice. The sacrifice had been the freedom which the third dimension offered—the freedom of the bird. Compared with this the freedom of the fixed wing seemed like that of the straight jacket. A necessary preliminary to becoming airborne was that run on the ground, and once in the air one could, like a dart, move only in the direction to which the nose pointed. Above all one was always compelled to fly fast for fear of stalling. Finally, on returning to earth, there was again that run before one could come to a standstill.

(*Flight*)

FIGURE 5. Cierva C.30 Autogiro.

Was this, then, the realisation of the age-old dream? Was there no better solution? Perhaps there was one but the road to it was winding and beset with obstacles by contrast to the straight and smooth avenue of the fixed wing! Only a fool it seemed, would waste time on a barren track which might perchance lead nowhere! However, the power of faith and conviction is never stronger than when challenged and credit must go to the men who met this challenge. I have in mind Cornu, Breguet (Fig. 4) (who achieved the first man-carrying helicopter)[4], Pescara, De Bothezat, Berliner, Baumauer, Florine, Ascanio and others who, despite and against the current of general technical opinion, continued their good work and thus, even though their practical success was only moderate, laid the foundations of a new branch in aviation. The years passed by, but progress was disappointingly slow—indeed, the helicopter became almost a butt for ridicule. It seemed to lack the power to develop and gather momentum for still further and more rapid advances—the power of the fire which the aeroplane so abundantly had shown—as though it were encumbered with an innate impediment dwarfing a healthy growth.

In this condition of apparent hesitation the rotating wing took a somewhat unexpected turn with the appearance, in the early twenties, of the "Autogiro," in which an auto-rotating rotor supplied the support during flight (Fig. 5). Its inventor—Juan de la Cierva—never believed in the idea of driving the rotor in flight[5], yet in the development of his aircraft made a most valuable contribution to the advancement of the helicopter. His foremost contribution consisted in the practical demonstration of the value of blade articulations, which permit flapping as well as inplane oscillations of the blade. Through this feature the Autogiro not only became the first rotating wing aircraft to achieve practical flight, but also showed the way to further helicopter development.

1.4. THE RENAISSANCE OF THE HELICOPTER

The success of the Autogiro is reflected in the Breguet-Dorand (Fig. 41) helicopter, which in 1936 took all helicopter records, and notably in the Focke-Achgelis helicopter (Fig. 6) which in 1937 set entirely new helicopter standards with the following world records[6]:—

Speed 123 km./hr. (76·43 m.p.h.)
Altitude 2,441 m. (8,009·4 ft.)
Duration 1 hour 20 min.

and which is generally accepted as being the first practical helicopter.

Two years later, in 1939, Sikorsky flew the VS.300 which marked the beginning of harvest time plus the production stage of the helicopter. Thereafter the rate of development accelerated.

The Second World War gave impetus to the helicopter but the real opportunity to show its suitability

FIGURE 6. Focke-Achgelis helicopter.

for use in military and naval operations came in the Korean war, where it played a major part in rescue operations on land and at sea, in transport and communication tasks in inaccessible territory, and in ambulance, observation and similar activities. Major civil roles of the helicopter became short haul mail, freight and some passenger transportation, crop dusting, geographical survey and similar work. The helicopter has achieved a speed of 236 km./hr. (146·64 m.p.h.), a height of 6,739 m. (22,111 ft.) and a distance in non-stop flight of 1,959 km. (1,217·3 miles)[6]. Other major achievements have been the crossing of the Atlantic by way of Greenland and Iceland, the crossing of the American Continent and, in Europe, circular flights extending a distance of about 5,000 km. (3,107 miles). The helicopter proved to be generally safer than the aeroplane, especially in bad weather, but its potential hazard was found to lie in its greater mechanical complexity. It turned out to be more expensive and much slower than the aeroplane.

In the meantime the fixed wing had achieved fantastic performances. After the Wright Brothers' early successes, Bleriot crossed the Channel in 1908—Alcock and Brown crossed the Atlantic in 1919. The U.S. Army made a flight round the world in 1924. Lindberg flew from New York to Paris non-stop in 1927. Today the records stand at:—

Speed 1,212 km./hr. (753·1 m.p.h.)
Height 19,406 m. (63,680 ft.)
Distance 18,082 km. (11,233 miles)

and Air Carrier-borne rocket aeroplanes have reached a speed of the order of Mach number 2·0 and heights close to 30,000 m. (98,430 ft.).

The fixed wing has become the paramount weapon of attack and defence. In the civil sphere air routes have sprung up all over the world and the aircraft has shown itself able to compete successfully with railways and ships. Thus, for example, on the trans-Atlantic route the passenger traffic in the air has lately exceeded that on sea. The exploitation of the fixed wing has indeed reached previously undreamt proportions.

However, as the picture developed so both light and shadow took shape and on the debit side there grew the hazard of speed coupled with power failure and bad visibility, the expense and complexity of ground organisation and equipment, and the inability to connect directly with the centres of human activity. The constitutional short-comings of the fixed wing had early been foreseen by aviators like Chanute, who wrote in 1894[2]:

> "A really adequate practical flying machine will hardly be said to have come into existence until it possesses the power of starting up into the air under all conditions. . . . Three principal methods have been experimented with: (1) by acquiring speed and momentum on the ground, (2) by the reaction of rotating screws, (3) by utilising the force of the wind. The first we have seen to require the use of special appliances such as railway tracks, so that its application must be limited, and the third necessitates that wind shall blow and with sufficient force. Either or both may be utilised with the earliest types of practical machines, *but the writer believes that the second method—that of rising through the reaction of a screw—will eventually supersede the two others.* . . . The alighting safely anywhere is also an unsolved problem, and one, as will readily be perceived without argument, of vital consequence. It has been slurred over by most of the designers of flying machines, and the best method which has been thus far proposed, involves the selection of a smooth, soft piece of ground and the alighting thereon at an acute angle. When it is considered that the speed required for support will be somewhere from 20-40 miles per hour it will be realised that the performance will be somewhat dangerous. . . ."

Little did Chanute realise though, that this aeroplane within a space of decades would grow into a metal monster of over 100 tons, that its alighting speed would be over 100 m.p.h. and the "smooth, soft piece of ground" would transform into a star of concrete avenues each over a mile in length, hundreds of feet in width and a yard thick.

The worry over the speed hazard of the fixed wing

FIGURE 7. Sikorsky helicopter in Korea.

found serious expression first in 1927 when the Guggenheim safe aircraft competition was announced, the declared object of which was "to achieve a real advance in the safety of flying through improvement in the aerodynamic characteristics of heavier-than-air-craft, without sacrificing the good practical qualities of present day aircraft." But the outcome of this competition was disappointing. It produced an improved wing-slot and an improved flap but it did not bring about a radical attack on the problem nor did it affect in any way the trend of aeroplane design towards higher runway speeds and longer runs[7].

The writing appeared on the wall, however, when Sir Frederick Handley Page in the Third Louis Bleriot Lecture, in May 1950[8], after discussing the size and cost of modern airports made the statement:

> "There is a limit to this big runway policy. Few countries are likely to be able to afford to extend many of the smaller runways, much less build many airports to such a standard, for not only are they expensive in money, but the amount of land immobilised is unjustifiable. An easement of aerodrome landing strip facilities would be welcomed, it is believed, by everyone—including the poor tax payer. At some airports extensions will prove to be physically impossible, and alternative sites, at others, will be so distant from the cities they are to serve, that aeroplanes *must* conform to present limitations or cities will have to forego the advantages that civil air transportation can confer.
>
> "While in civil air operations the installation of a first class runway may be impossible because of cost—particularly in some of the underdeveloped areas—in military operations the need for a super-long runway entails an immense expenditure of time and manpower before an operation can be carried out. Thus air power is deprived of that flexibility in strategy which is so essential a characteristic. This prodigal conception of runways has now to be countered. One of the main objects of this paper is to draw the attention of all concerned, operator, designer, and not least, the airport authorities, to those mounting capital and upkeep costs which must eventually be borne as part of the price for air operations, and then to review those aircraft design developments which would lead towards simplification of the basic problems of the airport and airline operators."

These exhortations of an aeroplane constructor squarely expose the Achilles heel of the fixed wing. But besides the runway argument there is that of the terminal time loss on the journeys between aerodrome and city centre.

The question of short-haul air transport has received increased attention lately, partly because of the alarming drop in recent years in the demand for this service, and partly in connection with the question of the DC-3 replacement.

The subject was discussed in some detail at the Sixth Annual Technical Conference of I.A.T.A. in 1953, which included a symposium on the operation and design requirements of helicopters[9]. On this occasion airline operators thought that a properly developed helicopter, giving a cruising speed of 115-120 knots (132·42-138·18 miles/hr.) and supported by an adequate ground organisation, could compete successfully with other forms of transport in a range of stage lengths from 30 to 200 miles and metropolitan operators regarded anything from 3 to 30 miles as suitable stage lengths. On the question of costs it was noted that helicopters of the present vintage were expensive in operation but design trends offered considerable improvement.

In the sphere of military operations the helicopter has become a *sine qua non*, from the large troop transport to the little ramjet-driven run-about with its colossal specific fuel consumption of 4·5 lb. per passenger mile!

This, then, is an outline of the birth and the early growth of the helicopter as well as the background against which this development took place. It brings us to the present and there now arises the question, "Whither will you go? And what can you do?"[10] Indeed what is the province, the domain of the helicopter?

1.5. DEFINITION OF HELICOPTER

It is the purpose of this paper in the first instance to discuss the fundamentals and limitations of the rotating wing, to survey a number of promising helicopter configurations, to discuss various physical (and especially aerodynamic) features, as well as design and development aspects peculiar to the helicopter, and finally to compare a number of typical helicopter and aeroplane operations. It is hoped that this study will indicate the fields of profitable employment of the helicopter as well as those tasks for which it is distinctly unsuited, and thus may suggest ways and means of its future integration into the general pattern of human affairs.

A necessary preliminary to this study is a definition of the word helicopter. Much has been written about and around the rotating wing, and this has produced a rather large glossary containing, inevitably, many nebulous or overlapping terms. Thus there are words like direct-lift machine, rotating-wing machine, thrust actuator orni-thopter, helicopter, helicoplane, helicogyro, Gyroplane, Gyrodyne, Autogiro, Convertiplane, Convertaplane, Convertible Aeroplane, Convertible Aircraft, Convertible helicopter, pure helicopter, compound helicopter and others. Some people think the "helicopter" term should mean: "an aircraft embodying wings, which are driven by a source of man-made power and rotate in a substantially horizontal plane, and which supply at all times during flight all aerodynamic forces necessary for support and control." From this narrowest definition of the helicopter the scope may widen to: "a flying machine designed to rise vertically by one or more lifting screws, revolving horizontally"[11] or simply: "an aircraft that can hover."

It is not an intention of this paper to resolve the dictionary confusion of the rotating wing nor to create new definitions for general purposes, but to establish only a few basic definitions for use in this paper.

> *Helicopter*—an aircraft, heavier than air, capable of sustained hovering, and embodying wings which, at least during the period of hovering, rotate in a free air stream and thus supply substantially the total lift for support.
>
> *Pure Helicopter*—a helicopter, the wings of which rotate in a substantially horizontal plane and,

throughout the flight, supply the aerodynamic forces for sustentation and propulsion.

Compound Helicopter—a helicopter, the rotating wings of which supply at all times during flight the major part of the lift, and which in addition embodies substantial aerodynamic components such as wings and/or propellers that, mainly at higher translational speeds, supplement the action of the rotating wing.

Convertible Helicopter—a helicopter, capable of conversion during flight such that the total lift is substantially transferred from the rotating wings to other wings and vice versa.

Notation

a { acceleration; lift curve slope $= \frac{dC_L}{d\alpha}$; blade cross sectional area at l
a_l blade cross sectional area at L
a_R blade cross sectional area at the rotor centre
a_{mean} mean value of cross sectional area between L and R
δa elemental increment of a (blade cross sectional area)
b number of blades
c blade chord
c_0 blade chord at the root
d distance between point P in the induced velocity field and vortex filament δs.
f stress
g acceleration due to gravity
h { height of rotor above the c.g. of the aircraft; height loss after partial power failure
l { moment arm; specific strength
$\delta l, dl$ elemental increments for l (specific strength)
l_1, l_2 horizontal lever arms for two rotor configurations
p rotor disc loading $= T/A$
r distance along the blade measured from the rotor centre
s rotor solidity $= \frac{\text{blade area}}{\text{disc area}} = \frac{bc}{\pi R}$
δs element of vortex filament
t time
t_1 time to reach the safety speed V_1
t/c thickness chord ratio of aerofoil
w blade weight/ft.^{2}
A { power required in hovering; rotor disc area
A_b total blade area $= A.s$
A_f flat plate area of the hub
B minimum power required in flight
C_L lift coefficient
$C_{L\,basic}$ mean blade lift coefficient for datum condition of flight (*i.e.* hovering)
$C_{L\psi}$ mean blade coefficient at azimuth angle ψ
$C_{L\,max}$ maximum lift coefficient obtainable before stalling occurs
C_m blade pitching moment coefficient
C_{Dp} blade profile drag coefficient
C cruising power
D total blade drag in the plane of rotation
D_L component of lift in the plane of rotation
D minimum power required for 200 ft./min. rate of climb
E power required for 600 ft./min. vertical climb
F { force; centrifugal force; value of engine power after partial power failure
$dF, \delta F$ elemental increments of F (centrifugal force)
H_a, H_e / H_r, H_g } amount of work done in time t_1
K exponential power
K_p ratio of required profile power in general flight to the basic profile power
L greatest length measured from the tip towards the rotor centre for which—with a given radius and tip speed—a uniform mass grading can be maintained
L_1, L_2 limit for c.g. traverse due to control movement limitation
L' rolling moment
M { moment; pitching moment
M_{crit} critical Mach number
N yawing moment
P power
P_p power to overcome blade profile drag in general flight
$P_{p\,basic}$ power to overcome blade profile drag in the basic condition of flight (*i.e.* hovering)
P_l centrifugal force at l
δP increment of centrifugal force
$\overline{Q}$ torque vector (Q = magnitude of $\overline{Q}$)
$\overline{Q}_1, \overline{Q}_2$ torque vectors in the two-rotor configurations
R rotor radius
T rotor thrust
$\overline{T}_1, \overline{T}_2$ thrust vectors in the two-rotor configurations
V forward flight speed
V_1 safety speed
V_a velocity perpendicular to the rotor disc
V_t velocity tangential to the rotor disc
V_T tip speed of the rotor $= R\Omega$
V_n velocity normal to the plane instantaneously containing the rotor blade
V_{eff} velocity in the plane instantaneously containing the rotor blade perpendicular to the longitudinal axis of the blade
$\overline{V}_r$ resultant velocity vector at the blade element dr
$|\overline{V}_r|$ magnitude of $\overline{V}_r$
$\overline{V}$ resultant relative velocity vector at blade element dr
V_i { induced velocity at a point P_i in the rotor velocity field; induced velocity
δV_i increment of V_i due to element of vortex δs
V_{crit} forward speed limitation due to Critical Mach number

W all up weight of aircraft
W_b weight of b blades
$W_{b\,min}$ minimum weight of b blades
W_h weight of the rotor hub
X longitudinal force
Y lateral force
Z normal force
α blade incidence
β_0 coning angle of the rotor
β_1 fundamental flapping angle of blades
ε_p blade profile drag/blade lift
ζ factor expressing variation of induced flow from forward to aft of the rotor
ι induced velocity, tip speed ratio $= \dfrac{V_i}{R\Omega}$
λ axial advance ratio of the rotor $= \dfrac{V_a}{R\Omega}$
μ tangential advance ratio of the rotor $= \dfrac{V_t}{R\Omega}$
μ_{max} maximum permissible advance ratio
ρ {air density; density of blade materials}
ρ_0 air density at sea level (standard)
σ relative density ratio $= \rho/\rho_0$
ϕ angle between axis of the vortex filament δs and d
χ angle between rotor thrust vector and the vertical
ψ azimuth angle measured from the aft position in the direction of rotation
Γ vortex strength
Ω angular velocity of the rotor

2. The Performance and Limitations of the Rotating Wing

2.1. GENERAL

The rotating wing differs from the fixed wing in so far as it performs a composite movement consisting of the rotary movement of the wing relative to the aircraft body and of the mainly linear movement of this body relative to the surrounding air. This combination of rotation and translation results in the general case in a substantial variation of air flow, not only from one point to another along the span of the wing, but also in a cyclic variation with time for any point along this span.

For this reason the aerodynamics of the rotating wing are more complex than those of the fixed wing, which is subject only to substantially linear movement. In other respects the aerodynamics of these two sorts of wing have much in common, especially principal aerodynamic phenomena of air flow and dynamic force, viscous flow and skin drag, circulation around the aerofoil and lift, its limitations and stalling, the air flow induced by lift and the consequent drag, and the effects of compressibility of air below critical speed and in the transonic range and the supersonic range. Another major feature of the rotating wing is the centripetal acceleration due to rotation. This acceleration is generally many hundred times that of gravitation and

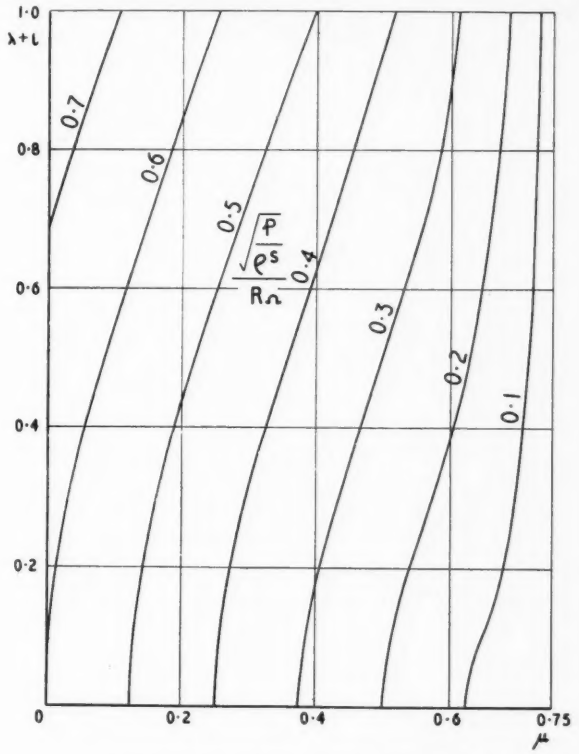


FIGURE 8. Rotor limitations—blade stalling.

produces by far the biggest force acting on the wing. The application of the foregoing aerodynamic principles to the composite movement of the rotating wing, together with the dynamic forces due to its rotation, form the basis of its performance as well as its limitations.

Rotating wings—or rotor blades—are arranged in a centrically symmetrical pattern or star shape—the Rotor—and are connected with the rotor hub either rigidly (apart from a limited freedom for rotation about the longitudinal blade axis in order to permit variation of pitch)—or they are given a greater angular freedom by means of additional articulations at the blade root. In the first case bending moments can be transmitted from one blade to another, or to the rotor hub axle and in the opposite case, that of full blade articulation, the bending moments at the rotor centre are zero. In all cases it is necessary, in order to avoid vibration, to keep the rotor axle tolerably free from fluctuating forces and moments, and this is best achieved by avoiding major lift and drag fluctuations of the blade.

2.2. AXIAL FORCES AND LIMITATIONS DUE TO BLADE STALLING

Considering lift first, the necessary freedom from fluctuations can be achieved in an air flow of a cyclic nature only if a suitable blade control, *i.e.* incidence variation during rotation is provided. This feature has been discussed in various papers[12, 13, 14] and is now text book knowledge[15]. The vector sum of the velocities due to rotation and translation is higher on

the advancing side of the rotor than on the retreating side and this would cause fluctuation of blade lift unless compensation was provided through a suitable variation of blade lift coefficient $C_{L\psi}$. If, for simplicity, the blade lift is assumed to be effectively concentrated at a point 0·75 rotor radius distant, then it can be shown[16] that this variation of $C_{L\psi}$, with azimuth angle ψ, can be expressed in the case of purely tangential flight (*i.e.* flight in the plane of the rotor disc) by

$$C_{L\psi}=C_{L\,basic}\Big/\left[1+\frac{4}{3}\mu\sin\psi\right]^2 \quad . \quad (1)$$

where ψ is the azimuth of the blade, μ is the advance ratio for tangential flight and $C_{L\,basic}$ the basic or mean lift coefficient of the blade in the datum condition of flight $[\mu=0,(\lambda+\iota)=0]$,

i.e. $$C_{L\,basic}=p/[\tfrac{1}{2}\rho s(\tfrac{3}{4}R\Omega)^2] \quad . \quad (2)$$

p being the rotor disc loading and s the rotor solidity (blade lift assumed at 0·75 R as before).

By the same reasoning, in the general flight case, which includes the axial velocity components $V_a=(\lambda+\iota)R\Omega$, equation (1) can be expanded to read

$$C_L=C_{L\,basic}\Big/\left[\left(1+\frac{4}{3}\mu\sin\psi\right)\sqrt{\left\{\left(1+\frac{4}{3}\mu\sin\psi\right)^2+\left(\frac{4}{3}\right)^2(\lambda+\iota)^2\right\}}\right] \quad . \quad . \quad (3)$$

It is evident that $C_{L\psi}$ becomes a maximum for $\psi=270°$ and if stalling is to be avoided at this point, $C_{L(\psi=270°)}$ must not exceed the $C_{L\,max}$ appropriate to the aerofoil at the Reynolds number employed,

i.e.

$$C_{L(\psi=270°)}=C_{L\,basic}\Big/\left[\left(1-\frac{4}{3}\mu\right)\sqrt{\left\{\left(1-\frac{4}{3}\mu\right)^2+\left(\frac{4}{3}\right)^2(\lambda+\iota)^2\right\}}\right]=C_{L\,max} \quad . \quad (4)$$

Assuming $C_{L\,max}=1{\cdot}3$ and substituting the expression in (2) for $C_{L\,basic}$, equation (4) after suitable arrangement takes the form

$$\frac{\sqrt{(p/\rho s)}}{R\Omega}=0{\cdot}53\,C_{L\,basic}=0{\cdot}6\sqrt{\left\{\left(1-\frac{4}{3}\mu\right)\left[\left(1-\frac{4}{3}\mu\right)^2+\left(\frac{4}{3}\right)^2(\lambda+\iota)^2\right]^{\frac{1}{2}}\right\}} \quad . \quad . \quad . \quad (5)$$

This relationship is plotted in Fig. 8 which gives blade stalling limitations in non-dimensional form.

It is evident that a high μ can be achieved only with a low $C_{L\,basic}$ and this in turn is obtained by a low blade loading p/s and/or a high tip speed $R\Omega$.

The equation (5) can be written as one in speed and this has been plotted in Figs. 9 and 10 giving limitation for axial and tangential flight respectively.

As density decreases with height, so the blade loading factor $\sqrt{(p/2\rho)}/R\Omega$ increases. The stalling limits increase with height, reducing the operating range of the rotor.

2.3. FORCES IN THE PLANE OF ROTATION AND LIMITATIONS DUE TO DRAG FLUCTUATIONS

A rotor blade is subject during rotation to radially acting (centrifugal) inertia forces and transversally acting lift forces, it thus assumes a position of equilibrium under these forces, expressed by the small coning angle β_0. This behaviour is obvious if the blade is articulated at the root, but this coning angle will also arise with a so-called rigidly mounted blade, because the bending stiffness of conventional rotor blades (except extremely small and stubby ones) is not sufficient to affect the powerful play of forces acting on the blade.

In oblique flight, the velocity of the air will have a component at right angles to the conical rotor disc which is larger aft than forward. Moreover, in forward flight the rotor lift causes an induced air flow at the rotor disc the velocity of which is slightly greater aft than forward[17]. These two factors produce a substantially larger inflow velocity aft than forward, and the inflow angle therefore varies accordingly. There is

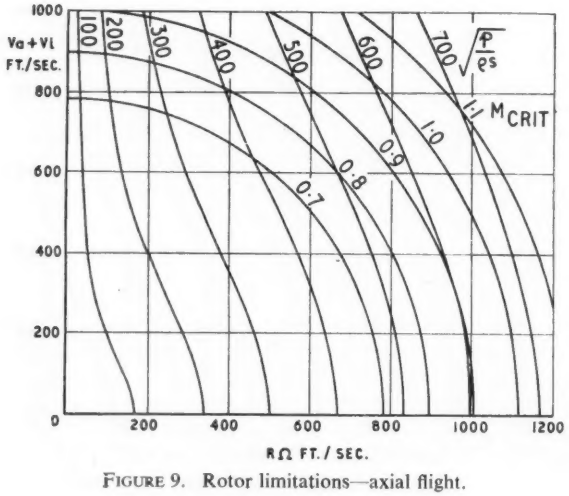


FIGURE 9. Rotor limitations—axial flight.

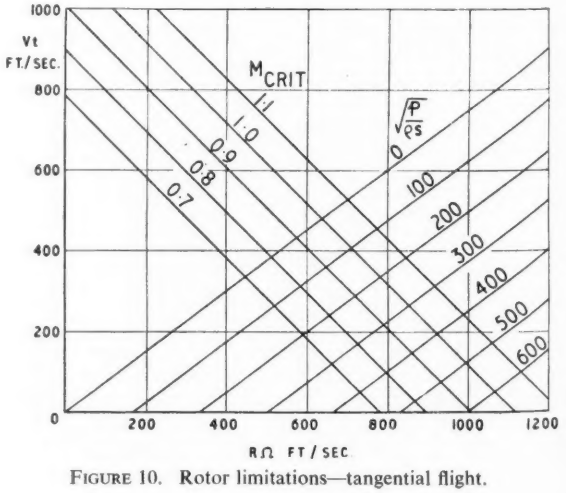


FIGURE 10. Rotor limitations—tangential flight.

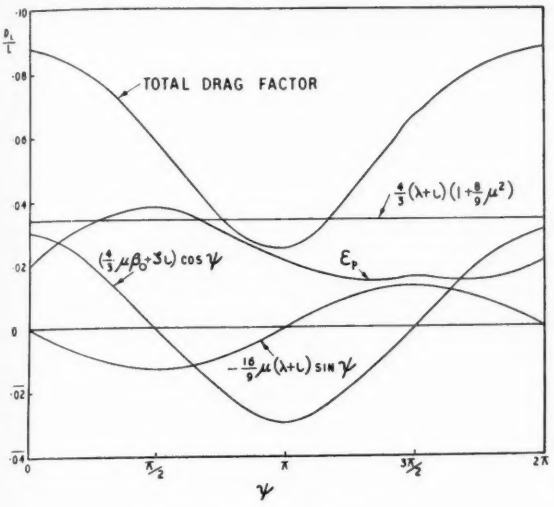


FIGURE 11. Drag fluctuation.

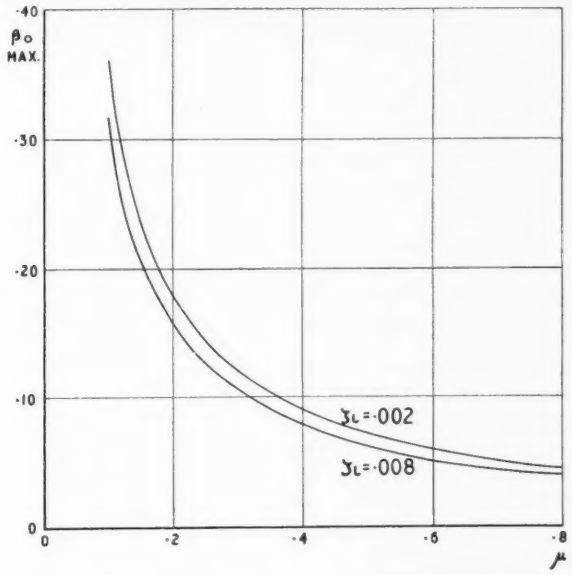


FIGURE 12. Rotor limitations—coning angle.

also a difference in the inflow angles at the lateral quadrants of the rotor, because of the difference of resultant blade velocity. This is large on the advancing side, which for a given inflow velocity tends to decrease the inflow angle. Exactly the opposite occurs on the retreating side of the rotor. As the blade lift remains substantially constant during rotation, the blade drag in the plane of rotation depends then mainly on the inflow angle. Moreover, there is the profile drag of the blade which is proportional to ε_p, which is high on the advancing side of the rotor blade and small on the retreating side (unless there is blade stalling). These factors have been considered in a simplified analysis[16] where the blade drag (taken at $0{\cdot}75R$) is shown as $D_L = LV_n/V_{eff}$, *i.e.*

$$D_L = L\left[\frac{\frac{4}{3}(\lambda+\iota)+\left(\frac{4}{3}\mu\beta_0+\zeta\iota\right)\cos\psi}{1+\frac{4}{3}\mu\sin\psi}\right], \qquad (6)$$

i.e. in terms of mean blade lift, which without much error can be written more simply as

$$D = L\left[\frac{4}{3}(\lambda+\iota)\left(1+\frac{8}{9}\mu^2\right)+\right.$$
$$+\left(\frac{4}{3}\mu\beta_0+\zeta\iota\right)\cos\psi-$$
$$-\frac{16}{9}(\lambda+\iota)\mu\sin\psi+$$
$$\left.+\varepsilon_p\right] \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where β_0 = coning angle

ζ = factor expressing variation of induced flow from fore to aft in the rotor

ε_p = profile drag/lift ratio.

Here the first term is independent of ψ. There is further a sin ψ term which together with ε_p generally adds up to a quantity which is also materially constant throughout rotation. The serious drag fluctuation appears to be the cos ψ term. This is shown in Fig. 11 which illustrates these terms for a flight condition of $\mu=0{\cdot}3$, $\lambda=0{\cdot}02$, $\beta_0=0{\cdot}065$ and $\zeta\iota=0{\cdot}0034$, that is to say, a typical helicopter in cruising flight. In this figure the cos ψ term variation is $\pm 0{\cdot}03$ blade lift, and this is a measure of the fluctuating aerodynamic forces in the plane of rotation. Assuming a limit of $\pm 0{\cdot}05L$ for these forces one can establish the maximum permissible coning angle.

$$\beta_{0\,max} = [0{\cdot}05 - \zeta\iota]\Big/\left(\frac{4}{3}\mu\right) \quad . \quad . \quad (8)$$

A plot of β_{0max} against μ for various values of $\zeta\iota$ is shown in Fig. 12 the product $\zeta\iota$ remains generally quite small throughout the speed range because $\lim_{V\to 0}\zeta=0$ and $\lim_{V\to\infty}\iota=0$, $\zeta\iota$ is a maximum in the so-called "transition stage."

Another significant vibratory force, acting in the plane of rotation, is connected with the kinematics of the flapping rotor blade[17]. In a rigidly held two-bladed rotor the magnitude of this force (without drag articulation) is

$$F = 2\frac{W_b}{g}R\beta_0\beta_1\Omega^2\sin(2\Omega t) \quad . \quad . \quad (9)$$

where W_b is rotor blade weight and β_0 and β_1, the coning angle and the fundamental flapping angle respectively, the latter of which is increasing directly proportionally with μ. With more blades in the rotor this force is reduced, but the argument remains materially the same. It is evident from these considerations that the maximum permissible coning angle must decrease as μ

increases. To alleviate these inertia forces, rotor blades are articulated to permit movement in the plane of rotation, or alternatively the whole rotor is mounted flexibly, to allow the same movement.

2.4. BLADE PITCHING MOMENTS AND LIMITATIONS DUE TO AIR COMPRESSIBILITY AND REVERSED FLOW

The third kind of rotor limitations are connected with undesirable moments about the longitudinal blade axis (pitching moments). As the major forces acting on the blade are lift and inertia, large moments will arise, if these forces, or components of them, produce a couple. Such a consequence is especially undesirable in the chordwise plane, for the blade is then twisted and its incidence altered. Great care is therefore taken in rotor blade design to avoid chordwise moments. Usually this is achieved by mass balancing (*i.e.* by arranging that the centre of gravity of all blade elements coincides with their aerodynamic centres, and lies on a substantially straight radial line) and in addition by choosing an aerofoil to give $C_m=0$ for this ($\frac{1}{4}$ chord) point. Generally this condition can be maintained throughout the whole subsonic speed range.

However, as soon as the critical speed is exceeded, considerable movement of the aerodynamic centre of the aerofoil takes place and this produces considerable pitching moments. Unless, therefore, the blade has been especially designed for this condition of flight, transonic operation, except perhaps for the extreme tip portion of the advancing blade, must be avoided. The critical speed V_{crit} of an aerofoil (the speed of drag divergence) depends on its thickness chord ratio and the lift coefficient[18]. In Fig. 13 the critical Mach number for symmetrical aerofoils is shown as a function of t/c for a range of values of C_L. This Mach number limitation arises mainly on the advancing side of the rotor, where the speeds from rotation and translation are added. Fortunately on this side C_L is a minimum. The advantages of low t/c ratio for the blade section near the tip and a low basic C_L are evident from this graph. In Fig. 14 various values for the speed ratio $V_{crit}/R\Omega$ are plotted against $(\lambda+\iota)$ and μ, from the foregoing consideration and the same limitations are shown in Figs. 9 and 10. These two graphs in particular show the maximum axial and tangential flight speeds that can be achieved, relative to the critical Mach number and the blade stalling limitation discussed earlier. The points of intersection of the two families of curves shown in each of these graphs represent speed maxima. Thus, for instance (Fig. 10), a blade with a critical Mach number of 0·9 and a blade loading factor $\sqrt{(p/\rho s)}$ (which has the dimension of speed) of 300 ft./sec., permits a maximum forward speed of slightly over 200 ft./sec., at a blade tip speed of $V_T=800$ ft./sec. With increasing height M_{crit} and ρ decrease, and this reduces the operating range of the rotor.

Another important rotor limitation connected with blade pitching moment is that due to the reversed air flow over the blade in a region of the retreating side of the rotor. The aerodynamic phenomena in or near this reversed flow region are rather complex. Three features, however, can be distinguished, all of which cause major

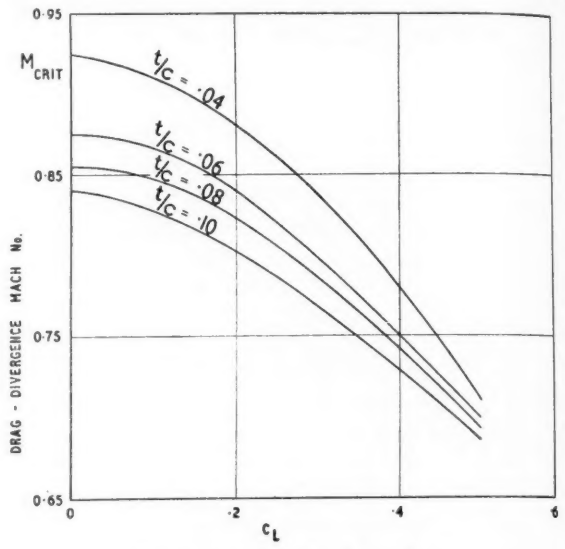


FIGURE 13. Critical Mach number.

blade pitching moments, and which vary according to the portion of blade passing through this region.

(*a*) pure blade stalling in the area surrounding the reversed flow region

(*b*) stalled flutter originating mainly within the reversed flow region, due to the negative lift slope of the reversed and stalled aerofoil.

(*c*) low incidence flutter or divergence in the central part of the reversed flow region. This arises

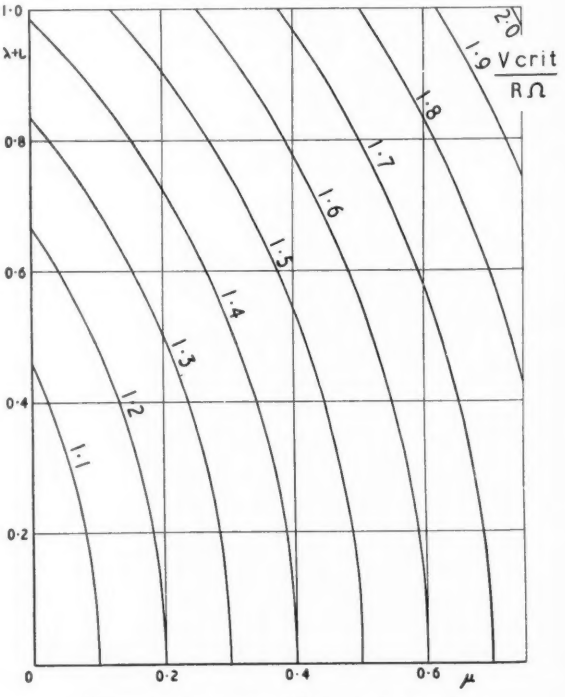


FIGURE 14. Rotor limitations—Mach number.

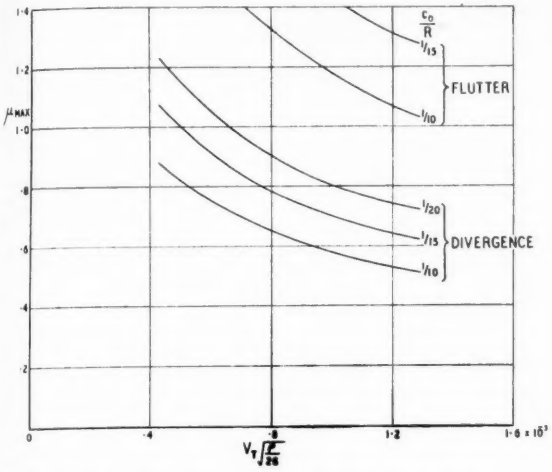


FIGURE 15. Rotor limitations—divergence and flutter.

because, with the flow reversed, the aerodynamic centre of the blade is moved to a point 0·75 chord distant from the normal leading edge. Thus the blade element, while satisfactorily mass-balanced in normal air flow, is very tail heavy in this reversed air flow. In this condition therefore the critical flutter and divergence speeds are very low.

Preliminary investigations have shown formidable mathematical difficulties in the theoretical treatment, mainly connected with blade torsional and flexural swinging modes in these complex conditions, and much work remains to be done before a more reliable appreciation of the problems can be obtained. In the meantime Fig. 15, which represents a blend of theory and empiricism, purports to give an envelope of limitation from the considerations under (*a*), (*b*) and (*c*). It will be seen that as the root chord c_0 and the tip speed V_T increase, the maximum permissible advance ratio μ_{max} decreases.

2.5. ROTOR PERFORMANCE AND POWER REQUIRED

The following paragraphs deal mainly with the factors which determine the power required by a rotor in different conditions of operation. The net climbing power of a rotor is the input power less the power required to overcome blade profile drag, induced drag from the rotor thrust and parasitic drag from the rotor hub.

The profile drag power is readily established in the near hovering condition of flight, which may be termed the "basic" flight condition, where there is no flow through the rotor, *i.e.* the advance ratios μ and $(\lambda+\iota)$ are nought. In this condition the rotor thrust is

$$T = C_{L\,\text{basic}} \frac{\rho}{2} b \int_0^R c\,(r\Omega)^2\,dr \quad . \quad . \quad (10)$$

where $C_{L\,\text{basic}}$ is the mean or "basic" lift coefficient, b the number of blades in the rotor and c the blade chord, which may vary with r. The basic profile drag power in this condition is

$$P_{p\,\text{basic}} = C_{Dp} \frac{\rho}{2} b \int_0^R c\,(r\Omega)^3\,dr \quad . \quad . \quad (11)$$

which for parallel or straight tapered blades (in which case c means the blade chord at $0{\cdot}75R$), can be written as

$$P_{p\,\text{basic}} = C_{Dp}\,\tfrac{1}{8}\,\rho\,A_b V_T^3 \quad . \quad . \quad (12)$$

where A_b = total blade area

and V_T = rotor tip speed.

The profile drag coefficient C_{Dp} depends on the t/c ratio of the aerofoil, the $C_{L\,\text{basic}}$ and the tip Mach number employed[19]. On these assumptions the basic profile drag power per sq. ft. blade area has been calculated and this is shown in Fig. 16 as a function of $C_{L\,\text{basic}}$ for various blade tip-speeds. In the general flight case the profile drag power is expressed by

$$P_p = \frac{1}{2\pi} b \frac{\rho}{2} \int_0^R \int_0^{2\pi} C_{Dp}\,c\,|V_r|^3\,dr d\psi \quad . \quad (13)$$

where $|V_r|$ is the scalar of the resultant speed vector $\overline{V_r}$ at the blade element dr. $\overline{V_r}$ is the sum of the vectors $\Omega \times \bar{r}$ and $\overline{V}$. The integral in equation (13) has been evaluated on the basis of certain simplifying assumptions which are valid for rotor blades of conventional design, and the result is shown in Fig. 17 where the profile drag power is expressed in terms of basic profile drag power (K_p) for a range of values of μ and $(\lambda+\iota)$. Thus in the general flight case the mean profile drag power per sq. ft. blade area is directly obtained by multiplying by K_p.

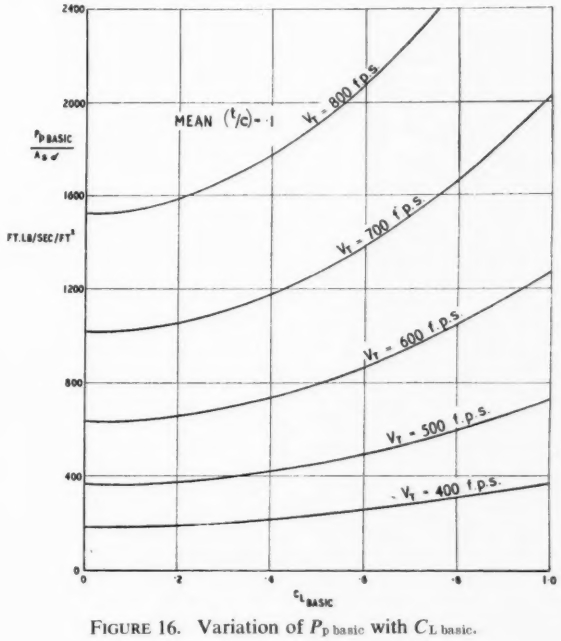


FIGURE 16. Variation of $P_{p\,\text{basic}}$ with $C_{L\,\text{basic}}$.

The induced power of a rotor is strictly a function of the air flow induced by the lift along the span of the individual rotor blades. Because of the composite movement of the blades the pattern of the induced air flow in the general flight case is rather complex. It varies not only from one point to another in the rotor disc, but for any point in the disc also varies with time, cyclically at blade frequency. This will readily be appreciated from reference to Fig. 18. Here is shown a simplified picture of the pattern of vortex sheets shed by the rotor blades. The vortex sheet of each blade is represented by two vortices which trail, one from a point near the blade root and the other from the blade tip. Any vortex filament of length δs and strength Γ produces at any given point P an induced velocity which has been found to be

$$\delta V_i = \frac{\Gamma \sin \phi}{4\pi d^2} \delta s \quad . \quad . \quad . \quad (14)$$

where d is the distance between P and δs and ϕ the angle between this line and the axis of the vortex filament δs. The total induced velocity at P is obtained by integrating equation (14) along all trailing vortices shed by the rotor blades. It is clear that the filaments nearer P have a greater share of the induced velocity at P than those further away and moreover, there will be a variation in V_i with time because of the down stream movement of the vortex lines relative to P. The rotor blades therefore experience major variations of inflow velocity which is, in fact, one of the sources of rotor vibrations and blade fatigue. The induced power for a blade element dr producing a thrust dT at P is $V_i\, dT$ and the total rotor induced power is then obtained by integrating with respect to ψ and r, which is a lengthy procedure. However, this can be simplified considerably by the assumption of an infinite number of blades, *i.e.* an actuator disc, which is now used generally for performance evaluation[15].

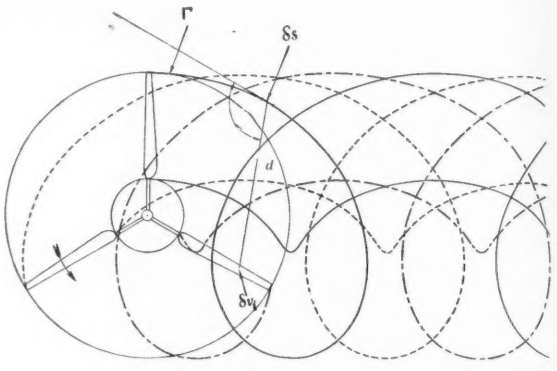


FIGURE 18. Vortex sheets shed by rotor blades.

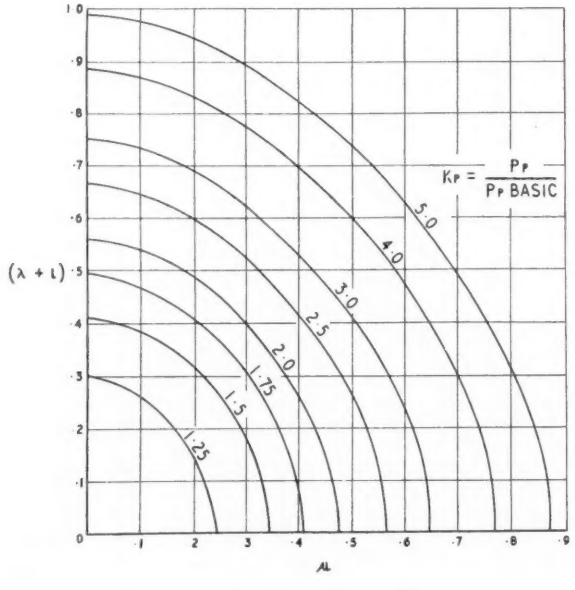


FIGURE 17. Power factor $K_p = \frac{P_p}{P_{p\ \text{basic}}}$

In the case of axial flight at high advance ratios this assumption leads to the momentum theory of the propeller by Rankine and Froude and, for tangential flight at high advance ratios, to the wing theory by Lancaster and Prandtl. Whereas in the axial flight case the *area* of the disc is the critical factor, in the tangential flight case the cross wind dimension or span is all important. For low advance ratios, however, both the theories break down and are replaced by semi-empirical relations[20]. It has been shown[17] that the mean induced velocity, the axial velocity V_a, the tangential velocity V_t and the disc loading factor $\sqrt{(p/\rho s)}$ are connected in a unique relationship which is shown in Fig. 19.

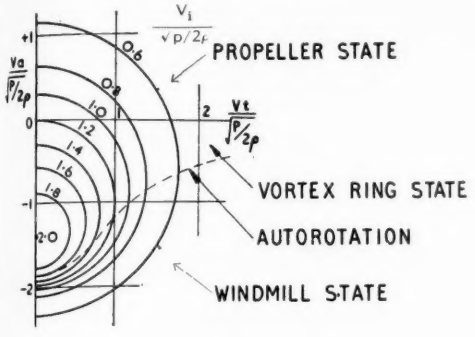
V_i represents the induced velocity or the induced power loss per lb. rotor thrust. The graph shows that with decreased advance ratio (μ and λ) there is a rapid increase of induced velocity, the maximum of which is approximately in the centre of the "vortex ring state" operating condition. Flying in this condition is generally unpleasant because of the turbulence produced by the high induced velocity. Allowance has been made in this graph for the different "wing spans" of some of the more common rotor configurations by the three scales for V_t. It will be noted that in forward flight the side-by-side rotor configuration has the least induced power loss. This is followed by the single rotor, and finally rotors in tandem have the highest induced power loss.

The power to overcome the drag of the rotor hub is obtained by assessing the flat plate area A_f of the hub. This depends greatly on design details and therefore a general expression is not possible.

2.6. WEIGHT OF ROTORS

The rotor consists of the rotor blades and the rotor hub. Its weight naturally depends, to a large extent, on design features but the purpose of this chapter is to show two fundamental parameters which control rotor weight and which apply to all rotors. One is due to structural requirements and the other is connected with the rotor coning angle β_0.

The structural considerations for blade weight are



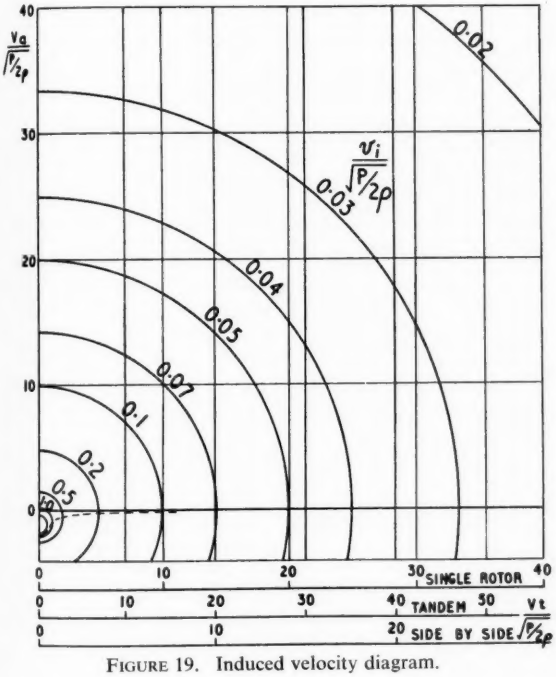


FIGURE 19. Induced velocity diagram.

based on a physical property of the building materials, namely the specific strength l,

$$l = f/(\rho g) \quad . \quad . \quad . \quad (15)$$

which expresses the ratio of stress over specific weight. It has the dimension of length and its physical meaning is as follows:—

A cylindrical body of length l made of a given material of density ρ, suspended at one end and subjected uniformly to one "g" acceleration, will experience a tensile stress f at the point of suspension. Elsewhere the stress will be less. If f corresponds to the maximum safe stress for centrifugal loads in rotor blade design, then for most of the materials used in rotor blades, l is around about 7,000 ft.

A similar length l can be established if such a cylindrical body is subjected to centripetal acceleration, such as in a rotor blade during rotation; in this event the argument goes as follows:—In a cylindrical body of unit cross-sectional area and of density μ, an element dl has the mass ρdl (*see* Fig. 20) and if the rotor of radius R rotates at speed Ω then the centrifugal inertia force acting on this element will be

$$dF = df = (R - l)\,\Omega^2 \rho dl \quad . \quad . \quad (16)$$

and therefore the tensile stress at this section from the centrifugal force of all the elements outboard thereof will be:

$$f = \rho\Omega^2 \int_0^l (R - l)\,dl = \rho V_T^2 \left[\frac{l}{R} - \frac{1}{2}\left(\frac{l}{R}\right)^2\right] \quad (17)$$

so that considering equation (15)

$$\frac{f}{\rho g} = 7{,}000 = \frac{V_T^2}{g}\left[\frac{L}{R} - \frac{1}{2}\left(\frac{L}{R}\right)^2\right] \quad . \quad (18)$$

This relationship, which is illustrated in Fig. 21, establishes a fundamental blade parameter, *i.e.* the greatest length L measured from the tip towards the rotor centre, for which in a blade of radius R and tip speed V_T a uniform radial mass grading can be maintained, the stress increases from the tip towards the centre and reaches at L the limiting value. Beyond this point the cumulative centrifugal force continues to increase and, therefore, as the stress may not be raised further, the cross-sectional area must be increased. The increase of blade cross-sectional areas inboard of L can be found as follows:—

A blade element δl (*see* Fig. 20) with cross-sectional area a is subject to the centrifugal inertia force of

$$\delta F = a(R - l)\,\Omega^2 \rho \delta l \quad . \quad . \quad . \quad (19)$$

but as on one side of the element the centrifugal force

$$P_l = fa$$

where f is now the maximum permissible stress and on the other side of the element

$$P_l + \delta P = f(a + \delta a)$$

$$\delta P = f\delta a = 7{,}000\,\rho g \delta a \quad . \quad (20)$$

hence $$\frac{da}{a} = \frac{V_T^2}{7{,}000g}\left(\frac{1}{R} - \frac{l}{R^2}\right) dl$$

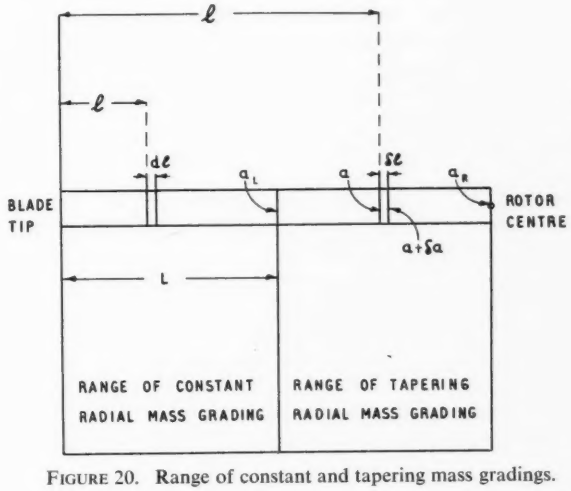


FIGURE 20. Range of constant and tapering mass gradings.

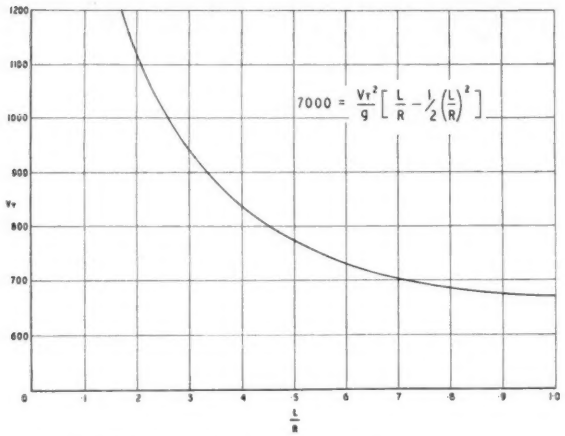


FIGURE 21. Lengths of constant radial mass grading.

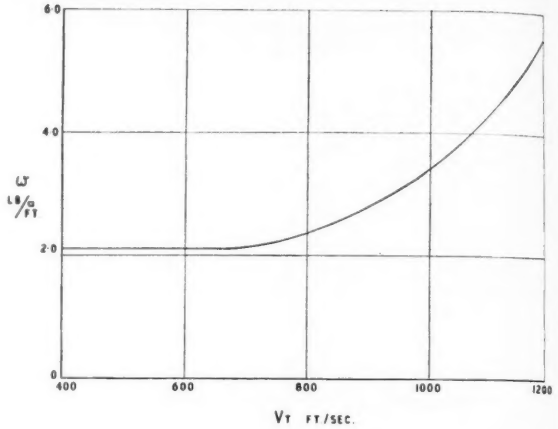


FIGURE 22. Blade weight.

and $$\frac{a}{a_L} = \exp\left\{\frac{V_T^2}{7{,}000g}\left[\frac{l-L}{R} - \frac{1}{2}\frac{l^2-L^2}{R^2}\right]\right\} . \quad (21)$$

where a_L is the cross sectional area at L, the point where it begins to increase, and a is the area at l. The maximum a_R is reached at the rotor centre. The mean value of a (in the blade portion from $l=L$ to $l=R$) is approximately

$$a_{\text{mean}} = a_L \exp\left\{\frac{V_T^2}{7{,}000g}\frac{1}{4}\left[1 - 2\frac{L}{R} + \left(\frac{L}{R}\right)^2\right]\right\} = a_L e^K \quad (22)$$

so that the minimum structural weight of the whole blade (the outer portion of constant radial mass grading plus the inner portion of tapering radial mass grading) is

$$W_{b_{\min}} = Ra_L\rho g\left[\frac{L}{R} + e^K\left(1 - \frac{L}{R}\right)\right] . \quad (23)$$

It has been found that the lightest panel construction in rotor blades weighs about 0·35 lb./ft.^{2} which makes the minimum basic weight of the blade 0·7 lb./ft.^{2} Such a blade would have its centre of gravity midway between the leading and trailing edge. In order to bring it forward to the quarter chord point, a weight of at least twice the basic weight must be added forward in the blade, making the minimum weight of balanced blade 2·1 lb./ft.^{2} Thus the minimum for $a_L\rho g$ is 2·1c and so one can write for a blade of constant chord (c)

$$\text{blade weight per ft.}^2 = w = 2{\cdot}1\left[\frac{L}{R} + e^K\left(1 - \frac{L}{R}\right)\right] \quad (24)$$

L/R is related to V_T in equation (18), thus w becomes a function of V_T only. This is shown in Fig. 22.

The second fundamental parameter controlling blade relates to the coning angle β_0. In Section 2.3 it was shown how during rotation the rotor blade assumes a position of equilibrium between the radially acting inertia forces and the transversely acting lift forces. Thus one may write (*see* Fig. 23).

$$\beta_0 \approx \tan\beta_0 = \frac{T - W_b}{F} \quad . \quad . \quad (25)$$

where T is total rotor thrust
W_b is total blade weight
and F is the centrifugal inertia force. Assuming the centre of gravity of the blade at $\frac{1}{2}R$, *i.e.*

$$F = \frac{W_b}{g}\frac{V_T^2}{2R} . \quad . \quad . \quad (26)$$

Alternatively, equation (26) can be written as follows:—

$$\frac{W_b}{T} = \frac{W_b}{W} = 1\Big/\left(1 + \frac{\beta_0 V_T^2}{2Rg}\right) \quad . \quad . \quad (27)$$

W_b/W being the ratio of blade weight over A.U.W. This equation shows a very important scale effect in the rotating wing, namely the larger the rotor the larger the ratio W_b/W.

One may now combine this rotor weight limitation, with that established in Section 2.3 where in equation (8)

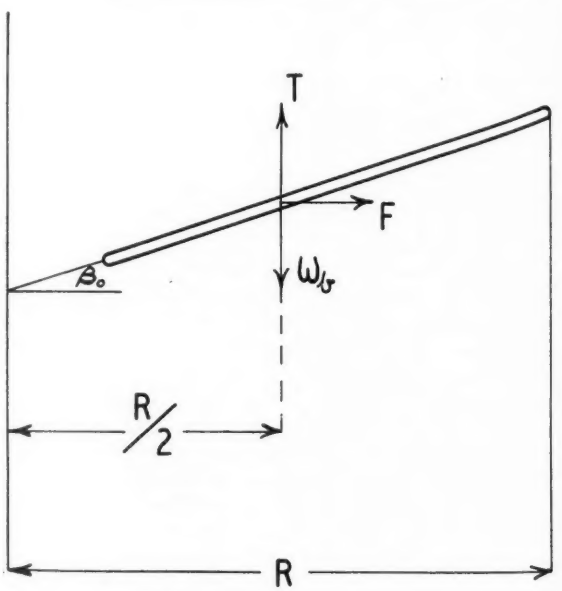


FIGURE 23. Blade forces.

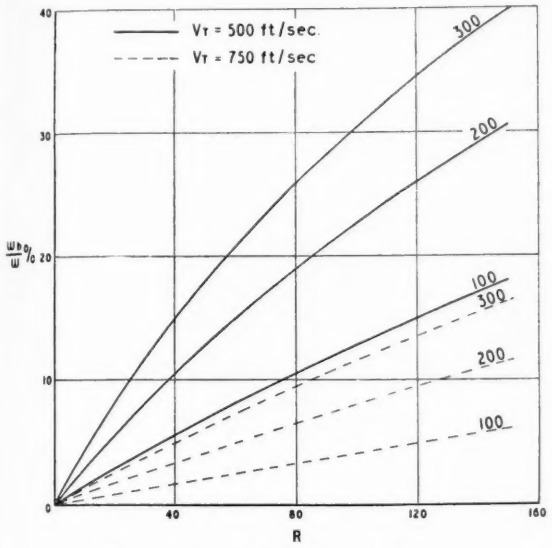


FIGURE 24. Weight of rotor blades.

an upper limit $\beta_0 = \beta_{0max}$ is established. Combining equation (8) and (27) gives

$$\frac{W_b}{W} = 1 \Big/ \left\{ 1 + \frac{3}{8} \frac{V_T^3 (0{\cdot}05 - \zeta_t)}{RgV} \right\} \quad . \quad (28)$$

This equation clearly shows the advantage of a high tip speed V_T. In Fig. 24 W_b/W is shown against R for various speeds V and tip speeds of 500 and 750 ft./sec. respectively.

Thus there are two independent parameters determining minima for rotor blade weight. The weight of small rotors is generally controlled by the structural limitation, and that of large ones by the coning angle.

The weight of the rotor hub is very much a function of design, generally, however, the following represents a fair estimate:—

$$W_h = W \frac{3{\cdot}8 \times 10^{-3}}{\beta_0} \quad . \quad . \quad . \quad (29)$$

3. Rotor Systems

3.1. GENERAL

The rotor system of a helicopter comprises the rotors, their hubs and control mechanism and, in the case of shaft-driven helicopters, that part of the transmission which joins the rotors together. The rotor system is driven by one or more power units through branches of the transmission, with a freewheel in each branch located, preferably, at the point of juncture with the rotor system.

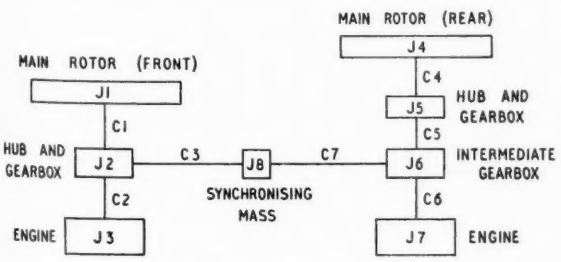


FIGURE 25. Rotor system of Bristol type 173.

The rotor system is *the* critical department in the helicopter. It has been said that the predominant hazard of the fixed wing lies in its high minimum flying speed which, combined with bad visibility or a power failure, is liable to lead to serious consequences. The hazard in the helicopter, on the other hand, is the mechanical complexity of the rotor system. In other engineering departments, the principle of redundancy is frequently employed in the interest of safety with good effect. Thus, for example, in the redundant structure of an airframe a broken member is automatically put out of use and the load taken over by others, or in the power system a faulty power unit is automatically disconnected at the freewheel and the work is transferred to other units.

There is, however, no collective safety in a number of rotors. We cannot afford to loose one rotor—or even one blade—and expect to continue to fly safely with the remainder. The rotor system is like a chain—if one link fails, the whole fails. Therefore, everything else being equal, the fewer rotors and blades there are in a rotor system, the greater is its safety: conversely, the greater the complexity of the system, the greater the chance of a failure. This is a fundamental truth in helicopter engineering.

As with the fixed wing (but perhaps in fuller measure), so the rotating wing is exposed to two types of loading: a limited number of applications of extreme

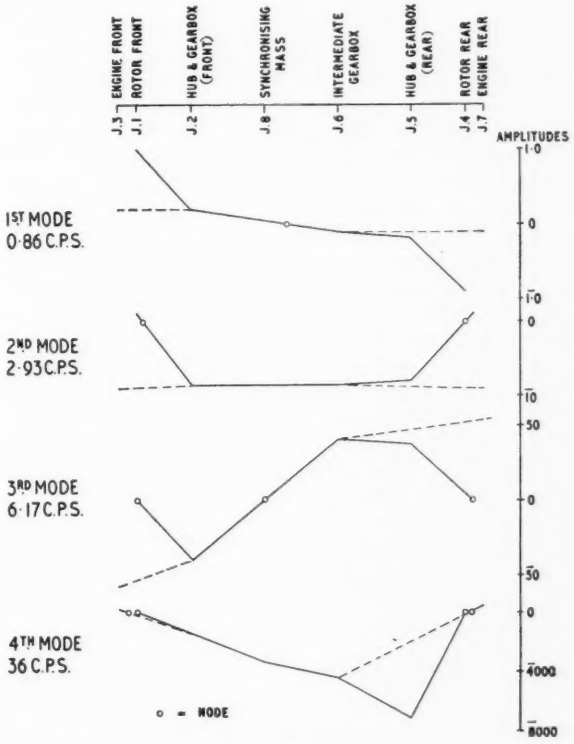


FIGURE 26. Type 173—swinging form diagrams of 4 modes.

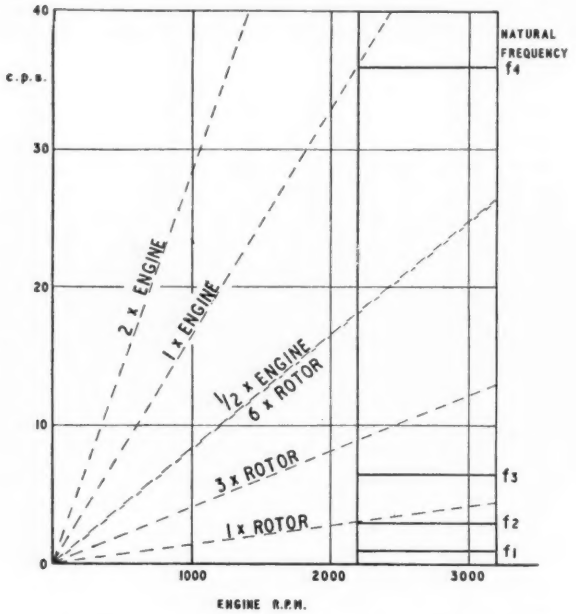


FIGURE 27. Natural frequencies of rotor system of Bristol type 173.

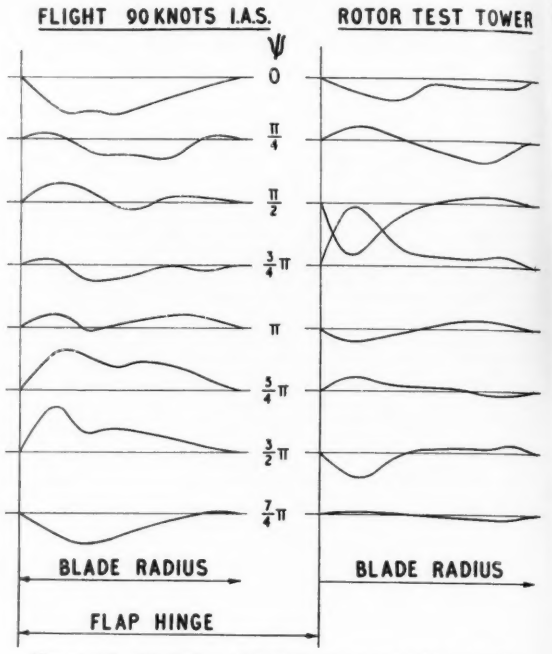


FIGURE 28. Variation of bending moment with azimuth.

load and frequent applications of medium load. It is thus necessary to establish on the one hand an envelope of ultimate load conditions where the material may be stressed close to the proof stress figure, and on the other hand to establish the so-called fatigue envelope, for which there is a much reduced safe stress level. The ultimate stresses arise generally from manoeuvre and gust loads, but as these are always coupled with inertia effects such as blade oscillations or torsional oscillations in the transmission, their evaluation is made somewhat difficult. Even more complex are the loading systems that produce the fatigue stresses, for these are due mainly to fluctuating air loads on the blades or torque fluctuation in the engines.

Two examples may illustrate this: Fig. 25 shows schematically the rotor system of the Bristol 173 helicopter together with the power units. It will be noted that this system consists essentially of concentrated masses which are connected by elasticities. It is therefore capable of swinging in a number of natural modes, a few of which are illustrated in Fig. 26. This rotor system is, in addition, subject to extraneous cyclic forces (from engine and rotor) and a condition of resonance will, therefore, exist whenever a forcing frequency is at, or near, that of a natural mode of swinging; high stresses are then experienced, particularly at or near the anti-nodes. Fig. 27 shows the natural frequencies of the Type 173's rotor system, together with those of excitations from the engine or the rotors. The points of intersection between these two families of curves represent points of resonance. Fig. 28 illustrates the bending moment variation in a blade during a revolution. This somewhat strange pattern is produced by a complicated aerodynamic loading system (particularly from the induced flow pattern shown in Fig. 18), on which is superimposed a system of dynamic loads, *i.e.* from simultaneously swinging in various flexural modes, essentially similar to that discussed in the previous example.

Besides the problem of stress in the rotor system, there is that of wear in bearings and other detail components. These considerations *in toto* have pointed to the inadequacy of purely theoretical work alone and have, in consequence, led to a procedure of ground testing in which the flight conditions are simulated with as much accuracy as is possible. Fig. 29 shows the Type 173 suspended on a gantry and undergoing an endurance run for such purpose. Similarly, in Fig. 30 a rotor is shown mounted on the rotor test tower at Bristol, where the flight loads are simulated by the introduction of suitable obstructions in the slipstream of the rotor. Fig. 31 defines the measured strains along the blade, obtained both with tower tests as well as flight tests. These examples have been chosen to demonstrate the difficulties which beset the designer in the development of the rotor system of a helicopter, and show the need for keeping this system as simple as possible.

3.2. ROTOR CONTROL

The rotor provides not only lift but also control of the helicopter. This must necessarily be so because the rotor blade is the only aerodynamic surface which maintains high speed in all flight conditions, including hovering, and is therefore capable of receiving aerodynamic forces adequate for the purpose of control. We must, however, distinguish between the rigid and the articulated rotor. At its hub, the rigid rotor can render a force—the rotor thrust—as well as three independent moments—the rotor torque and two transverse moments. On the other hand the articulated rotor can,

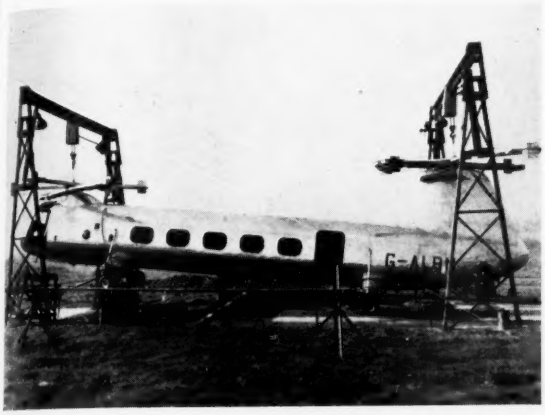


FIGURE 29. Bristol Type 173. Suspension in gantry.

apart from the rotor torque, produce three forces—the thrust and two transverse component forces. Rotation of the rotor axis produces in the rigid rotor a gyroscopic moment of considerable magnitude which is transmitted to the aircraft body. In the articulated rotor this gyroscopic moment is automatically balanced by an aerodynamic moment and there is no transverse moment transmitted through the rotor hub. Many of the so-called rigid rotors have blades which are, in fact, sufficiently flexible to permit typical flapping movement and thus in their action approach that of the articulated blade: conversely, articulated blades with large flapping-hinge offsets behave not unlike semi-rigid ones. The articulation of rotor blades has been found to overcome many of the difficulties which beset the early helicopters but, while at the same time it has also introduced difficulties of its own, such as in plane oscillations, since the successful performance of the Autogiro demonstrated the value of articulation it has been adopted almost universally in helicopter engineering. The following considerations apply therefore only to the articulated rotor:

Control through rotor torque. The rotor torque is a function of rotor speed and the mean blade incidence during rotation, *i.e.* collective pitch. As the rotor speed cannot be varied quickly enough for control purposes, variation of the collective pitch remains the only means of varying the torque. With this variation there is, however, an unavoidably connected major variation of rotor thrust. Such a form of rotor control does not, therefore, constitute a truly independent control, except in the case of two concentric rotors; this will be discussed later.

Control through rotor thrust. It has already been said that the rotor thrust is varied by the collective pitch of the rotor. If, then, the rotor speed is maintained constant, the collective pitch becomes an express control of rotor thrust. Such thrust variation is, however, associated with rotor torque variation—the argument being an inversion of the previous one—and consequently simultaneous adjustment by other suitable controls is necessary to avoid the undesirable by-products of control through thrust. The foregoing

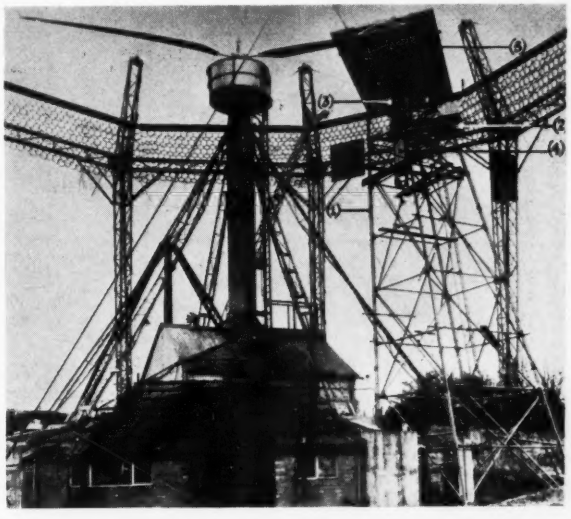


FIGURE 30. Rotor on Bristol rotor testing tower with obstruction. (1) Fixed structure (2) Working platform (3) Guide wheel cage (4) Column (5) Obstruction platform.

refers to the variation of the *magnitude* of the rotor thrust vector.

Control through rotor thrust angle. The rotor thrust vector acts substantially at right angles to the tip-path plane of the rotor, and tilting this plane in relation to the aircraft body will cause a corresponding tilt of the thrust vector. The tilting of the tip-path plane is effected by the cyclic or azimuth control.

Both collective pitch and azimuth control have become standard rotor controls and need no further comment here, but there is one fundamental observation that might well be made: the azimuth control, which is effected by operating on the rotor thrust vector, is dependent on the magnitude of this vector: if there is no thrust, there is no azimuth control. This feature is of fundamental importance in some rotor configurations and rotor-wing combinations.

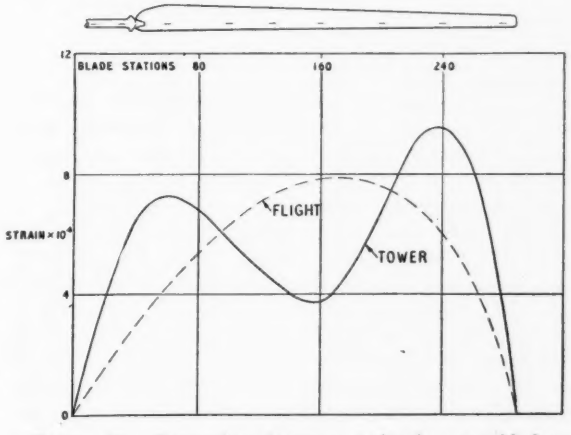


FIGURE 31. Comparison between strains in rotor blades measured in flight and on rotor tower.

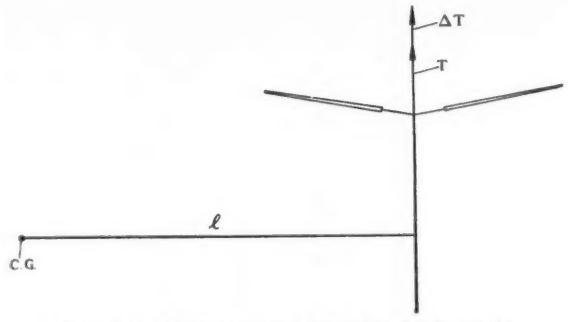


FIGURE 32. Moment control through collective pitch.

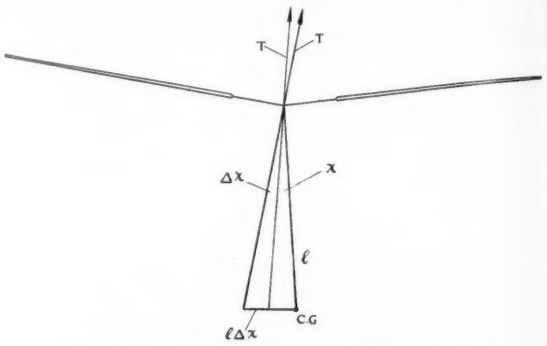


FIGURE 33. Moment control through cyclic pitch.

Even though the articulated rotor can, apart from rotor torque, only render forces at the rotor hub, it can nevertheless be employed effectively for the moment control of the helicopter. There are two cases:

(*a*) *Moment control through collective pitch*

If a rotor is located in a manner such that its thrust vector substantially passes through the centre of gravity of the aircraft, then a variation of this thrust will have no moment about this point. If, however, there is a considerable lever arm l of thrust vector about the c.g. (*see* Fig. 32), then a change of thrust ΔT will be accompanied by a considerable change of moment.

$$\Delta M = l\Delta T \quad . \quad . \quad . \quad (30)$$

(*b*) *Moment control through cyclic pitch*

A similar argument applies to this case. If a rotor hub is placed at or near the centre of gravity of the aircraft, then a tilt of the tip-path plane relative to the aircraft body will produce only a change in the balance of forces; it will not produce a material change in the moments. But if the rotor hub is placed at a considerable distance l from the c.g. (*see* Fig. 33) then a change $\Delta\chi$ in the direction of the rotor thrust vector will produce a substantial moment about this point.

$$\Delta M = Tl\Delta\chi \quad . \quad . \quad . \quad (31)$$

These fundamental control considerations are the basis of the control configurations which are discussed later.

3.3. TRANSMISSION OF POWER

Power transmission may take many forms and even in helicopter engineering a variety of proposals have been made. These range from the orthodox mechanical transmission to "gas shafts," between on the one hand, a gas generator and a jet in the tip of a rotor blade, or on the other hand, a gas turbine converting the gas power into shaft power. Nevertheless the author is convinced that the mechanical transmission will stay with the helicopter for some time to come, simply because, for all but the largest rotors, it is the lightest and most efficient form by which power can be supplied in the form needed at the rotor. The following therefore refers to power transmissions consisting of speed converting gearboxes and shafting. The engineering philosophy for such transmissions is similar to that of electrical power transmission. In the latter, heavy currents necessitate heavy conductors but very high voltages produce problems of insulation. In the mechanical transmission, similarly, high torque incurs heavy shafting, whereas high speed of rotation necessitates large ball bearings and poses the problems of shaft whirling. Investigations have shown that the optimum transmission speed for power shafting is between 3,000 and 4,000 r.p.m. To achieve the lightest transmission of power it is therefore necessary in the first instance to convert engine speed to the optimum transmission speed in a gearbox close to the engine or, preferably, directly attached to it. This applies with greater force to turbines because of their very high rotational speeds. At the other end of the power transmission—the rotor head—a second speed conversion which is made in another gearbox, preferably forming one assembly with the rotor head reduces transmission speed to rotor speed. While there may be additional gearboxes in the transmission system (for example, at the junctions of power lines and similar points) it is obvious that in the interests both of safety and weight saving it is the simplest possible layout involving a minimum of gearing and shafting that is wanted.

In Fig. 34 an arrangement of an optimum power transmission is indicated schematically, and in Figs. 35 and 36 the specific weights for gearboxes and shafting respectively are shown. The graphs are based on the foregoing assumptions and, in particular, on a working life of 7,500 hours.

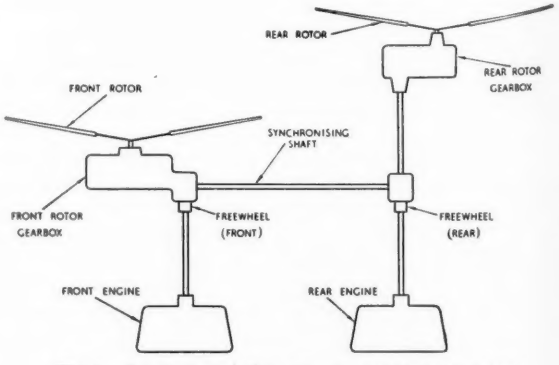


FIGURE 34. Transmission system of Bristol Type 173 (schematic).

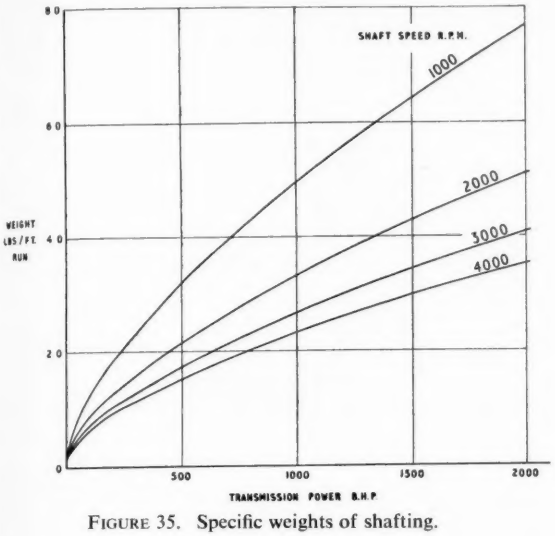


FIGURE 35. Specific weights of shafting.

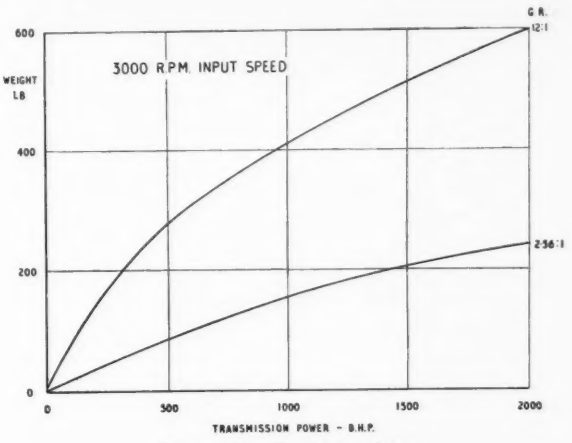


FIGURE 36. Gearbox weights.

3.4. ROTOR CONFIGURATIONS

In the following survey of the various rotor configurations attention is focused in the first instance on the need for the balance of forces and moments on the aircraft, secondly on its control and stability under various conditions of loading, thirdly on certain aerodynamic considerations relating to performance, fourthly on weight and finally, on the complexity of the transmission and similar factors. It is not possible here to enter into a quantitative analysis of these configurations (so much depends on detail of design) but merely to point out the major trends.

We must at the outset distinguish between the tip-driven and the shaft-driven rotor. The tip-driven rotor is torque balanced within itself and only rotor thrust is transmitted to the aircraft body. The thrust vector of such a rotor can be made to balance the weight of the aircraft if the rotor is placed direcly above the centre of gravity, and this simple arrangement achieves the full equilibrium of forces and moments. As regards control (Fig. 37), varying the magnitude of the rotor thrust vector produces (1) a vertical control force; a tilt of this vector in the longitudinal plane produces (2) a force in this plane; and a lateral tilt (3) produces a lateral force. If the hub is placed at a suitable distance l above the aircraft c.g. then the longitudinal and lateral components of tilt of the rotor thrust vector will also produce (4) a pitching and (5) a rolling moment respectively. There remains only the sixth component of three-dimensional movement, the yawing moment. In a small helicopter this can be obtained conveniently by placing a control surface in the form of a rudder within the slip stream of the rotor (Fig. 38).

Driving at the blade tip therefore permits the construction of a true single-rotor helicopter, indeed it is the only practical form of single-rotor helicopter. In respect of c.g. travel, however, the single-rotor helicopter is necessarily rather limited. If the maximum rotor tilt is χ then the c.g. must lie within the limits L_1 and L_2 (Fig. 37). In practice, the angle χ is generally limited to about 0·1 radians, so that, unless l is unusually large, there is a serious restriction. The tip-driven rotor has, nevertheless, a marked advantage on the score of weight. It is shown in Section 2.6, that the weight of a rotor blade is controlled by the coning angle, and large rotor blades particularly must be mass balanced at their tips. This parasitic weight can be avoided if the power unit is placed in the same position; the effective weight of this unit is then reduced by the amount of ballast which would otherwise have to be carried. It can, indeed, be argued that the effective weight of power units can be reduced to nought in very large rotors[21].

A rotor by itself is about neutrally stable; a single rotor helicopter can hence achieve a moderate degree of stability—at least in forward flight—if the fuselage is suitably shaped and horizontal and vertical surfaces are fitted at the tail. Apart from the feature of weight saving in large rotors, the single tip-driven rotor is attractive because of its mechanical simplicity.

Just as tip drive is natural to the single-rotor helicopter, so mechanical drive is natural to the multi-rotor helicopter. This conclusion is supported by the following reasoning:

(*a*) In Section 2 it was shown that rotors are subject to limiting flight conditions. From this it is obvious that a multi-rotor helicopter will achieve the widest flight envelope when its

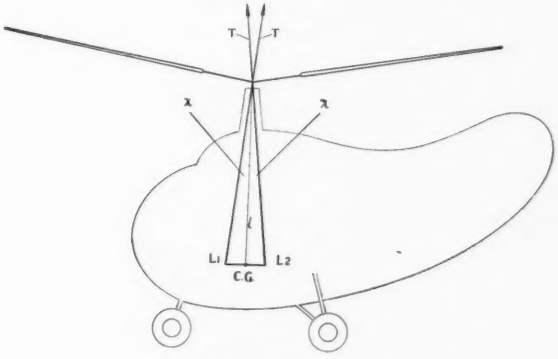


FIGURE 37. C.G. range.

rotors are so devised that these limiting conditions are reached by all simultaneously.

(*b*) This is best achieved by turning the rotors at a rate such that their tip speed is the same; that is, large rotors turn slowly, small rotors spin fast and rotors of equal size rotate at the same speed.

(*c*) This is best achieved by a mechanical interconnection in the form of shafting and suitable gearing.

(*d*) Another reason for maintaining a constant speed-relation between the rotors is the control of the aircraft. It has been shown that the simplest rotor control is based on a variation of blade incidence, with the speed kept substantially constant. As in a multi-rotor helicopter all rotors are utilised for control purposes, the maintenance of constant speed in the whole rotor system is most desirable. There is little room for argument that this is best achieved by the interconnection of the rotors through a mechanical transmission.

(*e*) Such a transmission can be linked, at little additional expense in weight and complexity, with one or more sources of power.

In shaft-driven multi-rotor configurations the balance of forces and moments is more complex than in the single rotor. This can be demonstrated by citing the general case of a two-rotor helicopter. Fig. 39 shows schematically the forces and moments in such a configuration, where $\overline{W}$ represents the weight of the aircraft and $\overline{T}_1$, $\overline{T}_2$ and $\overline{Q}_1$, $\overline{Q}_2$ are respectively the thrust and torque vectors of the two rotors. Vertical and horizontal components of these vectors are denoted by the suffices V and H respectively. For the balance of forces it is necessary that

$$\overline{T}_1+\overline{T}_2+\overline{W}=0 \quad . \quad . \quad . \quad (32)$$

This vector equation represents a triangle in a vertical plane, as shown in Fig. 39. T_{1V} and T_{2V} add up to the weight of the aircraft and $T_{1H}=T_{2H}$. For the balance of moments (about 0) it is necessary that

$$\overline{Q}_1+\overline{Q}_2+\overline{T}_1\times\overline{l}_1+\overline{T}_2\times\overline{l}_2=0 \quad . \quad . \quad (33)$$

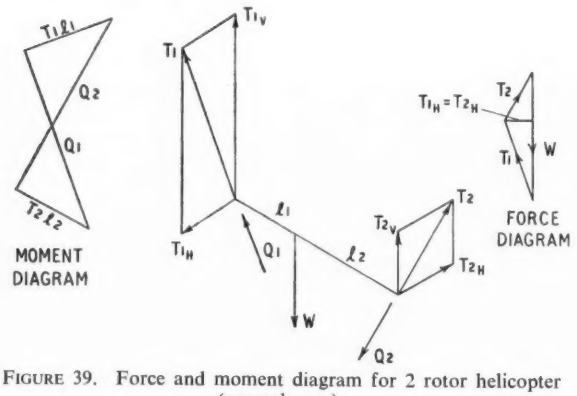


FIGURE 39. Force and moment diagram for 2 rotor helicopter (general case).

FIGURE 38. Hiller Hornet.

In this vector equation $\overline{Q}_1$ and $\overline{Q}_2$ must lie in vertical planes and by suitable choice of the origin 0 (as in Fig. 39) all four vectors can be made to lie in one vertical plane. Clearly an infinite number of combinations with these parameters is possible, but all these combinations fall into two groups:

(*a*) Torque-balanced or twin-rotors. These are rotors with torque vectors $\overline{Q}_1$, $\overline{Q}_2$ which add up to zero.

(*b*) Others.

From the latter group only one form has come into prominence (Fig. 40). It consists of one large rotor supporting the whole weight of the aircraft ($T_{1V}=W$ and Q_1 is large) and a small rotor aft of the main rotor, producing a relatively small lateral thrust T_2 which, in conjunction with a large lever arm l_2, balances the main rotor torque Q_1. There are small additional forces and moments necessary to complete the balance and, therefore, this configuration conforms to the general picture shown in Fig. 39. It is evident that the force and moment diagrams are unsymmetrical and rather complicated. In particular a small change in any of the

FIGURE 40. Sikorsky S-56 transport helicopter.

forces or moments, such as is caused by normal disturbances in flight, requires adjustment almost all round and, as a result, this type of helicopter is more difficult to fly, both for human and automatic pilots, than are the symmetrical forms of aircraft.

The necessarily different speeds of the unequal rotors produce a wider band of excitation frequencies than applies with either a single rotor or equal rotors, and this increases the difficulties of avoiding resonant conditions. On the score of rotor weight there is also a penalty, because two rotors of different size weigh more than two equal rotors capable of carrying the same weight. This can be deducted directly from the formulae in Section 2. As regards safety, the arrangement is a retrograde step relative to the true single rotor because of the introduction of a small second rotor and the consequent transmission system, but it is probably slightly better than the other two-rotor configurations, because experience has shown that a failure of the tail rotor, or its transmission branch, does not necessarily lead to fatal consequences. In many other respects this configuration is rather similar to the single rotor (especially in control in pitch and roll and the respective c.g. limits).

There are various forms of torque-balanced twin rotors. Besides being equal in torque they are generally identical in other respects (lift, diameter, speed) but they rotate in opposite directions. We distinguish between eclipsed and non-eclipsed rotors. Eclipsed rotors are either concentric (Fig. 41), or they are closely intermeshing (Fig. 42). The balance of forces and moments in this configuration is like that of the single rotor, except that in the former case torque (and thrust) can be increased in one rotor and decreased in the other (the total rotor thrust remaining constant), so producing a net rotor torque which can be utilised for yawing control. Compared with the single-rotor helicopter there is, however, the structural expense of two rotors, with no material aerodynamic gain and, in particular, no gain in effective rotor disc area. There is also the danger of blade fouling in gusty conditions. Nevertheless, on the asset side it should be noted that eclipsed rotors permit a more compact transmission system than do the other two-rotor configurations.

FIGURE 41. Bréguet-Dorand coaxial helicopter (1936).

FIGURE 42. Kaman HOK-1 helicopter.

There are two forms of non-eclipsed twin-rotors and both have been tried. The first is the side-by-side arrangement (Fig. 6) and the second is the arrangement in tandem (Fig. 43). In both configurations the rotors have the full benefit of the total disc area in the hovering condition, but as a *quid pro quo*, the transmission system is more extensive and heavy than that involved in eclipsed rotors. The balance of forces and moments is elementary (Fig. 44).

$$\left.\begin{aligned} &\overline{T}_1=\overline{T}_2;\ \ \overline{T}_1+\overline{T}_2+\overline{W}=0 \\ &\overline{Q}_1=\overline{Q}_2 \\ &\overline{l}_1=\overline{l}_2;\ \ \overline{T}_1\times\overline{l}_1=\overline{T}_2\times\overline{l}_2 \end{aligned}\right\} \quad . \quad . \quad (34)$$

It will be seen that, while these configurations, like all twin-rotor configurations, are symmetrical with respect to the normal direction of flight, the control is substantially different from that discussed previously. There is now an "axis of suspension" connecting the two rotor hubs and the thrust vectors $\overline{T}_1$ and $\overline{T}_2$. The centre of gravity of the aircraft lies at a distance h below this axis.

The following refers specifically to the tandem rotor arrangement, but the same argument applies to the side-by-side arrangement if the axes are suitably changed:

A lateral tilt of the thrust vectors $\overline{T}_1$ and $\overline{T}_2$ in the same direction causes the aircraft to experience a side force

$$Y=(T_1+T_2)\sin\chi \quad . \quad . \quad . \quad (35)$$

and a rolling moment

$$L'=(T_1+T_2)\,h\sin\chi \quad . \quad . \quad . \quad (36)$$

similar to that of a single rotor.

A lateral tilt of the same vector in opposite directions produces a yawing moment

$$N=(T_1 l_1+T_2 l_2)\sin\chi \quad . \quad . \quad (37)$$

A tilt of these vectors in the fore and aft direction produces a force

$$X=(T_1+T_2)\sin\chi \quad . \quad . \quad . \quad (38)$$

FIGURE 43. Piasecki H-16 tandem helicopter.

and a pitching moment

$$M=(T_1+T_2)\,h\sin\chi \quad . \quad . \quad . \quad (39)$$

similar to that of a single rotor.

A simultaneous increase of rotor thrust ΔT on both rotors through the collective pitch control causes a normal force,

$$Z=2\Delta T \quad . \quad . \quad . \quad . \quad . \quad (40)$$

whereas an increase $+\Delta T$ on one rotor together with a decrease $-\Delta T$ on the other (differential collective pitch control), produces a pitching moment (in addition to that in (39)),

$$M=(l_1+l_2)\,\Delta T \quad . \quad . \quad . \quad (41)$$

This is a substantial moment because of the magnitude of (l_1+l_2), a feature which permits a considerable c.g. travel along the axis of suspension without seriously upsetting the balance of the aircraft. This represents a major advantage for the tandem helicopter, over the other configuration, particularly as it can be arranged that the "axis of suspension" runs with the major axis of the fuselage; in this event substantial movement of passengers (or freight) along this axis can take place with little consequence.

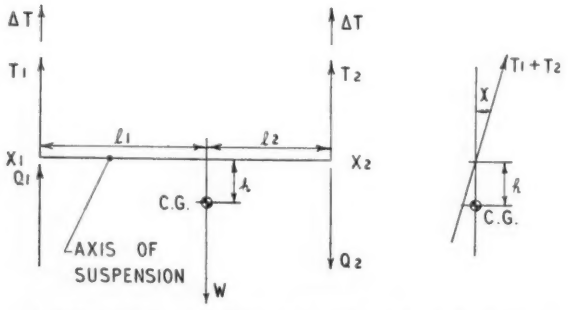


FIGURE 44. Force and moment diagrams for tandem and side-by-side rotors.

While the aerodynamic performance in hovering is the same, there is a significant difference in forward flight between the side-by-side and the tandem configuration. The side-by-side configuration has a large "wing span" over the two rotors and consequently its induced losses are relatively low, whereas the tandem configuration, with only half the span of the side-by-side, has a higher induced flow. Against this, allowance must be made for the considerable weight and the parasitic drag of the lateral outrigger structure which side-by-side rotors require, whereas in the tandem arrangement the rotor supports are light and have negligible drag.

Configurations involving more than two rotors are not discussed here. The author believes that the increased complexity and the consequently reduced safety standard of such elaborations are not acceptable. They can offer no material advantages over the two-rotor configurations.

3.5. POWER SUPPLY

The general requirements for power plants are common knowledge, and the following observations are, therefore, confined to power-plant features that are specific to the helicopter. The power units and that portion of the transmission on the engine side of the freewheel, or its equivalent, do not strictly belong to the rotor system: because of their interdependence with it, they have, however, been included appropriately in this Section.

In the first instance there arises the general question of the power required in a helicopter. This varies considerably, not only with the basic types of helicopter (discussed in Section 4) but also with the various rotor configurations and, too, with the design parameters chosen. There are, however, general trends common to all which are now discussed.

Figure 45 refers to a pure helicopter of moderate cruising speed, where the power required for level flight (S.L.) is shown against speed, power being expressed as a percentage of that needed for hovering. The engine power needed to give a suitable rate of climb in critical conditions of flight is also indicated. Point A represents the power required to hover. The minimum cruising power at speed V_{opt} is B, which, in conventional designs, is about $0{\cdot}5A$. The cruising power at V_{max} is C, which is roughly equal to $0{\cdot}75A$. The minimum power to give a rate of climb of 200 ft. per min. is D, which is about equal to $0{\cdot}57A$, and the power to give a vertical rate of climb of about 600 ft. per min. is E, which is approximately $1{\cdot}12A$. For a single-engined helicopter a take-off rating is selected which, at the least, meets Point E, and the cruising rating C is then established in relation to E by:

$$\frac{C}{E}=\frac{0{\cdot}75A}{1{\cdot}12A}=0{\cdot}67, \quad . \quad . \quad (42)$$

a figure which is generally suitable for engines.

For aircraft powered by two or more engines it is a general requirement that safe continuation of flight shall be possible after the failure of one engine at any time during flight.

Thus in the twin-engined helicopter the power D must be achievable by one engine at a continuous power or, at least, one hour power rating, to meet the contingency of an engine failure *en route*.

An engine failure during hovering is a more difficult problem. It might be desirable to reach point E with one engine on the basis of a take-off rating, or even possibly a slightly higher rating, because this power would be required only for a matter of seconds in a rare emergency. It is, however, improbable that an engine could bridge the gap between D and E, and the best that might reasonably be achieved is (Fig. 45):

$$F = 1{\cdot}25D \quad . \quad . \quad . \quad (43)$$

At this power the aircraft can maintain height only at a speed above V_1 and might be said to be in danger after take-off in a built-up area until it reaches V_1. This danger can be avoided by a suitable take-off and landing procedure which has been developed in conjunction with the design of the Bristol Type 173. According to this procedure (Fig. 46), the helicopter operating in a built-up area takes off along a flight path which is inclined 60 degrees to the horizontal in a *backward* direction. Should an engine failure occur during the take-off, the aircraft will sink back to the take-off point as there is not sufficient power to maintain height. It has been demonstrated that the sinking speed is not high and the undercarriage can cope with it. Once a given height, the so called "safety height," is reached, then the aircraft can leave the inclined flight path and assume *en route* flight. Should an engine failure occur just on reaching the safety height or thereafter, the pilot will not return to the take-off point but will try to attain as quickly as possible the safe flying speed V_1; in doing so he will lose height. The safety height must obviously be such that it exceeds by a requisite margin the unavoidable height loss. This height loss h can be calculated as follows:

The power required during this critical period is dropping from A approximately linearly as speed increases, *i.e*:

$$P = A\left(1 - \frac{V}{100}\right), \quad . \quad . \quad (44)$$

and the safety speed is then (*see* Fig. 45):

$$V_1 = 100\left(1 - \frac{F}{A}\right). \quad . \quad . \quad (45)$$

The aircraft is assumed to have constant acceleration throughout this emergency, so that

$$t = \frac{V}{a} \text{ and } t_1 = \frac{V_1}{a} = \frac{100}{a}\left(1 - \frac{F}{A}\right) \quad . \quad (46)$$

where t_1 is the time to reach V_1.
Thus the total work required for sustentation during the time t_1 with loss of height is:

$$H_r = \int_0^{t_1} A\left(1 - \frac{at}{100}\right) dt = \frac{100}{2a} A\left[1 - \left(\frac{F}{A}\right)^2\right] \quad (47)$$

the work to accelerate the aircraft mass to V_1 is:

$$H_a = \frac{W}{g}\frac{V_1^2}{2} = \frac{W}{2g}100^2\left(1 - \frac{F}{A}\right)^2 \quad . \quad (48)$$

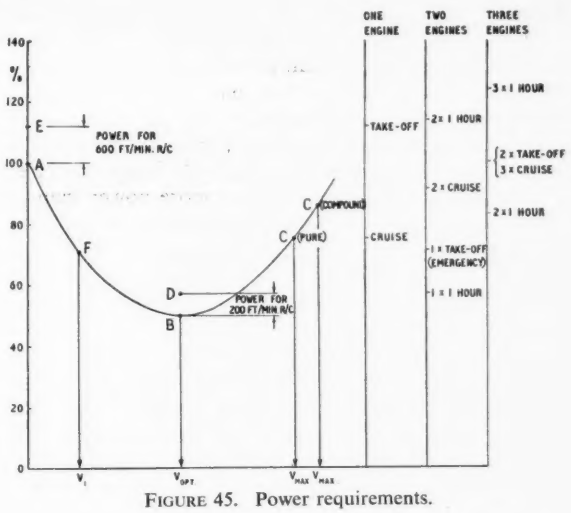


FIGURE 45. Power requirements.

The total work available from the engine during t_1 is:

$$H_e = Ft_1 = \frac{100}{a}F\left(1 - \frac{F}{A}\right), \quad . \quad (49)$$

and the work done by gravity due to the loss of height h is:

$$H_g = Wh \quad . \quad . \quad . \quad (50)$$

Hence the equation of work done in time t_1 is:

$$H_e + H_g - H_r - H_a = 0 \quad . \quad . \quad (51)$$

which, with the expressions in the equations from (47) to (50), gives:

$$h = \frac{100^2}{2g}\left(1 - \frac{F}{A}\right)^2\left(\frac{A}{W}\frac{g}{100a} + 1\right) \quad . \quad (52)$$

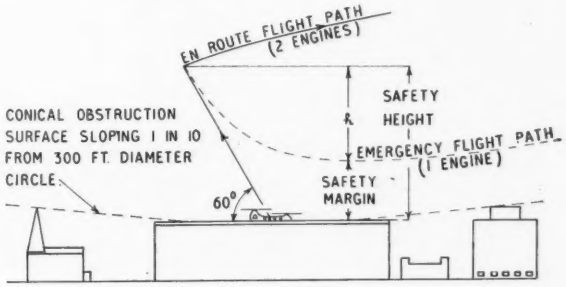
It will be seen that h decreases as F increases and

$$\lim_{F \to A} h = 0 \quad . \quad . \quad . \quad (53)$$

h is moreover dependent on the power loading W/A and the acceleration a.

The latter is limited by control considerations to about $0{\cdot}1g$, and on this basis the height loss h is plotted in Fig. 47 against F/A for various values of W/A.

If the twin-engined helicopter is designed to hover on one engine it is rather over-powered for normal operation, which adversely affects its economics. This,


EN ROUTE FLIGHT PATH (2 ENGINES)
SAFETY & HEIGHT
CONICAL OBSTRUCTION SURFACE SLOPING 1 IN 10 FROM 300 FT. DIAMETER CIRCLE
EMERGENCY FLIGHT PATH (1 ENGINE)
SAFETY MARGIN
60°
FIGURE 46. Backward take-off procedure.

however, does not apply, to the same extent, if there are more than two engines (Fig. 45). Thus, for instance, a three-engined helicopter can be designed to reach point *A* with two engines only at take-off power, and the same engines at maximum continuous power or one hour power will safely exceed point *D*, and this satisfies the *en route* engine failure requirement. Although the three engines at cruising rating will give more power than at point *C*, the excess will not be detrimental to fuel consumption. The same engines at high power will exceed point *E*. With more than three engines the limiting design condition is entirely that for hovering with one engine inoperative.

Similar arguments apply to the compound helicopter. As this type of helicopter can achieve a higher V_{max} than the pure helicopter, the point *C* will be somewhat raised in relation to the other points. This alters the above arguments quantitatively but not qualitatively.

An entirely different picture is presented by a certain group of convertible helicopters which will be discussed in Section 4. For these aircraft very high rotor disc loadings are advantageous and thus the necessary hovering power is considerably increased, so raising point *A* in relation to the others. In this case, moreover, at least four engines are necessary, with the ability to hover on three.

There are various types of power unit in use in helicopters. Those used for tip drive are:

(*i*) The ram-jet (Fig. 38) and the pulse-jet. Both these engines are extremely light, partly intrinsically and partly because they replace blade ballast. On the other hand their fuel consumption is very high.

(*ii*) The pressure jet (Fig. 48). This has a lower fuel consumption but is heavier and more complex in its installation.

(*iii*) Turbo-jets. These units have been proposed for installation at the rotor blade tip, not only for large slow-rotating rotors[21], but also for smaller ones[22]. Such an arrangement combines the advantages of low weight, simplicity and low fuel consumption, but the main difficulties are in the high centrifugal and gyroscopic forces to which the engines are exposed.

The power forms for shaft-drive are:

(*i*) The piston engine combined with a clutch. This power unit is economic in consumption but is heavy and involves complexity in installation. The power characteristics of the piston engine do not, unfortunately, suit the helicopter and this incompatibility is aggravated in multi-engined types.

FIGURE 48. Ariel II.

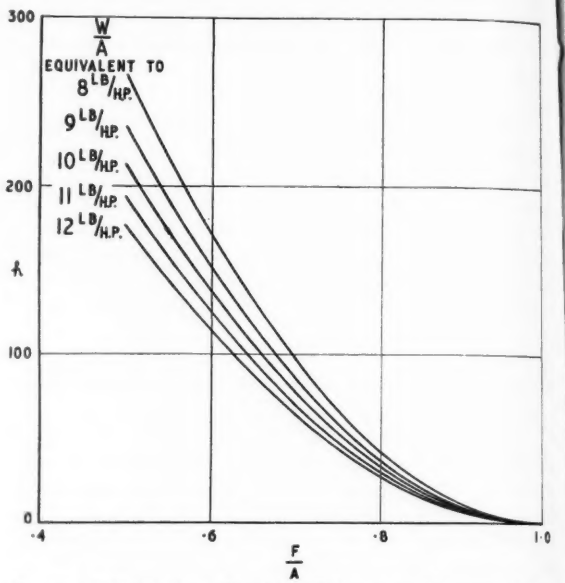


FIGURE 47. Height lost after take-off due to partial power failure.

In normal cruising flight the rotors require to run fast and the engines are happiest at comfortable cruising speed, but in the one-engine-out case it is desirable to lower the rotor speed in order to reduce the demand on power and, at the same time, to increase the speed of the operating engines to make up for the loss in power. These conflicting desiderata can be fully satisfied only by a two-speed gearbox. Studies and, indeed, experiments have been made on this problem: they have shown that such a gearbox is impracticable.

(*ii*) The free gas turbine. This engine has higher fuel consumption than the piston engine, but it is lighter and its installation is much simpler. Its main advantage, however, lies in the fact that the only driving connection between the power producer (the gas generator) and the rotor-driving turbine is the gas stream. Not only does this eliminate the clutch of the piston engine but it also serves as an infinitely variable gear reduction over a useful range of speed ratios, without appreciable loss in efficiency. Thus the free turbine succeeds where the piston engine fails, *i.e.* in the provision of power and speed characteristics which suit the rotor. In Fig. 49 are shown the weight of complete power installation (including gearboxes and shafting where necessary) and of the fuel consumed for various types of power plant. The specific weight per horse power (maximum continued power rating) is plotted against the duration of flight.

It is evident that the light tip jets have the advantage in short range flying, whereas the elaborate but more economic installations are superior for the longer ranges.

4. The Three Basic Types of Helicopter

In Section 1 three basic types of helicopter were defined:

(*a*) the pure helicopter
(*b*) the compound helicopter
(*c*) the convertible helicopter

This Section describes these helicopters, portrays their distinctive features and gives some indication of how they may best come into their own.

4.1. THE PURE HELICOPTER

This type of helicopter relies wholly on its rotors for sustentation as well as propulsion. The weight-carrying rotor, or rotors, remain substantially horizontal and the axial component of air flow $R\Omega\ (\lambda+\iota)$ is never very large. The critical limitations of the rotors are in the tip speed and the tangential advance ratio μ. Fig. 13 shows that for a symmetrical aerofoil of t/c ratio 0·06, which is just possible structurally at the blade tip, a Mach number of 0·87 can be achieved at low C_L without development of shock, and Fig. 10 gives the maximum forward speed, as a function of tip speed, which this Mach number would permit (at S.L.). This figure also shows the low-speed limitation resulting from the stalling of the retreating blade; this depends on the important parameter $C_{L\,basic}$, which can also be expressed as blade loading factor $\sqrt{(p/\rho s)}/(R\Omega)$. A low tip-speed can be achieved by a low blade-load p/s (but it should be noted that a low disc loading by itself does *not* control this limit). There is, however, a practical limit to the reduction in blade loading, as will be seen later, but which in this case is about $\sqrt{(p/\rho s)}=200$.

The rotor limitations, therefore, are a triangle, defined by the two limit lines and the base, the latter being the tip-speed range at hovering. The intersection of the two limit lines, the apex of the triangle, represents the limiting condition where the speed range of the rotor has shrunk to zero. This point indicates the maximum permissible speed of the helicopter as well as the corresponding optimum tip-speed. It is evident that a wide base of the triangle is the means of providing a high cruising speed. The practicable maximum cruising speed lies slightly below the apex and for the typical pure helicopter, is of the order of 247 ft./sec.=146 kts., at S.L.

Besides this there are other limitations which must be considered. Fig. 12 gives the relation between μ_{max} and β_{0max}, the latter term being connected with blade weight. In addition there is the flutter criterion for the reversed flow region, which permits only low μ values for large root chords and high tip speeds. The flutter criterion, however, will seldom affect the pure helicopter rotor. Another limit is given in equation (28) and Fig. 24, showing that the blade weight assumes an increasing proportion of the all-up weight if the rotor radius or the forward speed increases; on the other hand

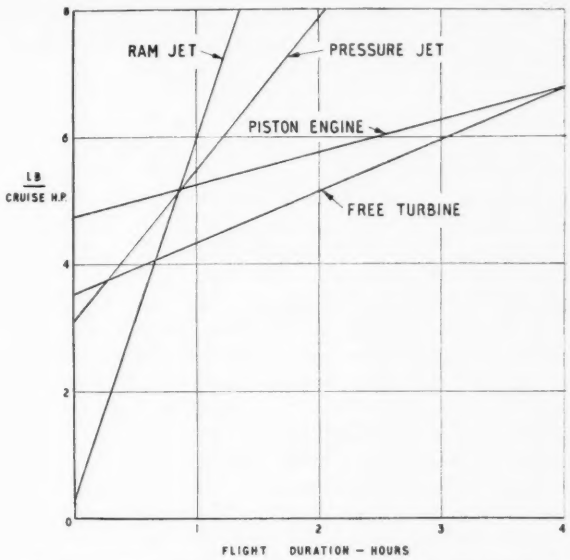


FIGURE 49. Total weight of power plant installation and fuel consumed per cruise h.p.

an increase in tip-speed is advantageous. Consideration of rotor radius leads to the matter of disc loading. A high disc loading corresponds to a small rotor, giving a small W_b/W. Alternatively, or additionally, this ratio can be decreased effectively by the use of two non-eclipsed rotors instead of one, the optimum arrangement obviously being twin rotors. High disc loading has repercussions on the performance of the aircraft. In Fig. 19 it can be seen that the disc loading factor $\sqrt{(p/\rho s)}$ controls the induced power, which is critical in the hovering condition. The other performance factor is blade loading. Lowering the blade loading (or $C_{L\,basic}$), desirable though it may be for the extension of the operating limits of the rotor, increases the profile power in hovering (Fig. 16) and therefore in translational flight as well (Fig. 17). Finally, there is the structural consideration for rotor weight which, in particular, affects small rotors of high solidity and tip-speed (Fig. 22).

It will be seen from the foregoing that compromises must be made between the conflicting desiderata and much depends, therefore, on the specification against which the project is made. Thus for example, a single-engined helicopter must have a satisfactory auto-rotating performance to permit forced landings to be made. This requirement restricts the disc-loading ($p<3{\cdot}0$ lb./ft.^{2}) and, to some extent, the blade loading and the tip-speed. In multi-engined helicopters, however, where this argument does not arise, the optimum disc-loading is of the order of 6·0 lb./ft.^{2}

Summing up, it can be said that the speed envelope of the pure helicopter is controlled mainly by, (*a*) the design of the blade tip and, (*b*) the blade loading, and speed increases beyond certain limits are bought at progressive expense in power.

4.2. THE COMPOUND HELICOPTER

There are many forms of compounding the rotor with other aerodynamic devices to advance the

performance of the helicopter, but by far the most effective means is the stub wing (Fig. 50). Such a wing has no effect at hovering or during slow-speed flight, but as speed is gained it supplements progressively the rotor thrust until, at a given maximum speed, a worth-while percentage of the all-up weight is taken by the wing, the rotor being correspondingly unloaded. The effect and the value of this action can be studied by reference to the previous example of the pure helicopter. If it is now assumed that a given stub wing is capable of carrying one half of the A.U.W. of the pure helicopter, at its maximum design speed, then the blade loading factor is reduced in this flight condition to half the previous limiting value. This means that the low-speed limit point in Fig. 10 at this design speed is moved to the left, while the base point of the line remains unchanged. This makes the low-speed limit line steeper. The important result of this change is a shift of the apex of the envelope triangle to the left and *upwards*. Thus the speed of the helicopter is increased without change either in the rotor blade area or rotor profile power. The rotor coning angle is also reduced and this diminishes vibration; alternatively it will permit a reduction in blade weight.

On the other hand, if no speed increase is desired, the stub wing can be used to enhance the economy of the helicopter, as it then permits an *increase* of the blade loading factor by two, without ill effects, but with the advantage of a reduction in profile power. Allowance must be made, on the debit side, for the drag and the weight of the fixed wing. There is, however, a certain net gain, because the fixed wing is more efficient in the production of lift at speed than is the rotating wing, and by mounting the undercarriage to the wing a considerable weight saving is achieved.

There are, nevertheless, certain limits which must be observed in the unloading of the rotor. The helicopter is basically controlled in flight through its rotor, or rotors, and a loss in rotor thrust means a loss in some of its control (*see* Section 3). If, therefore, the rotor unloading assumes considerable proportions, it is necessary for the rotor control to be supplemented by a wing control.

In addition, there is the problem of propulsion. In a pure helicopter sustentation and propulsion are obtained together by a small forward tilt of the rotor vector. Compounding the rotor with a stub wing will, however, decrease the rotor lift while at the same time increasing the aircraft drag. This means that the rotor thrust vector must be given a considerable forward inclination which, at a certain stage, begins to introduce aerodynamic complications. These difficulties may to some extent be delayed by additional compounding with a propeller or with a turbo-jet. A high degree of rotor unloading may sometimes be combined with a reduction of rotor speed to reduce rotor profile power, but this produces high advance ratios μ and, with them, the danger of reversed flow flutter. The limitation defined in Fig. 15 must then be considered.

FIGURE 50. Bristol Type 173 with stub wings.

4.3. THE CONVERTIBLE HELICOPTER

Although enthusiastic supporters of the convertible helicopter have been active for some time—and one is reminded in this connection of Herrick's courageous efforts back in the early thirties[23]—serious endeavours in this field with a chance of practical success are a comparatively recent development. The reason for this may be that some of the better and more advanced studies on the subject have revealed a more realistic and interesting picture than the superficial proposals made earlier. Something else which may possibly have had its effect is the fact that there have lately been indications of official interest in this form of flying machine[24, 25]. It is true to say that the convertible helicopter is typical of that type of engineering which provides a happy hunting ground for habitual inventors and that rather more patent recipes have been offered than is good for digestion. There is, indeed, wide scope for imagination and invention among the many possible forms of convertible helicopter, but despite this diversity one can distinguish only between two major types of convertible helicopter and two forms of conversion:

Type 1: The rotating wing is transformed during conversion into a fixed wing and sustains at all times the weight of the aircraft.

Type 2: The rotating wing is released during conversion from its function as a support of the aircraft, and this function is transferred to a fixed wing.

Form 1: The conversion does not involve major linear and angular accelerations, *i.e.* it consists of a succession of quasi-static flight conditions.

Form 2: All other conversions.

A critical appreciation of the various examples of convertible designs and the forms of conversion could easily fill a book. There is evidence of interdependence between the factors controlling the performance of the conversion and those connected with the economics and the handling of the aircraft generally. It is therefore not possible to obtain a conclusive picture in any one aspect without reference to the other. Indeed, it is not possible to arrive at a practical appreciation of the possibility of a convertible helicopter without making a relatively detailed study both of its design and its operation. Such a study cannot be made here, but a few conclusions may

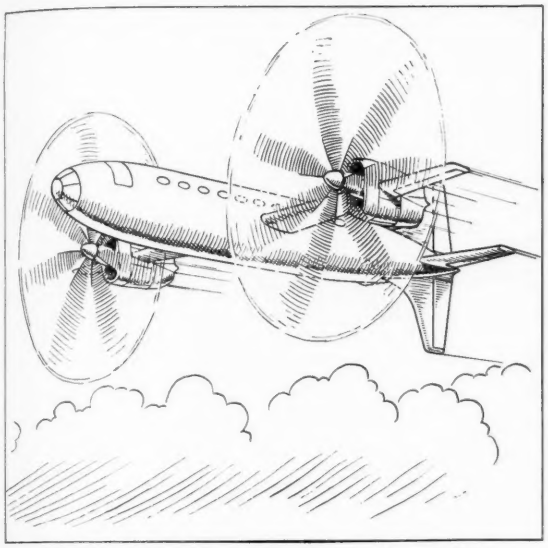


FIGURE 51. Convertible helicopter in horizontal flight.

be presented which are the result both of a survey of the field and preliminary studies of certain features.

Briefly, then, it may be stated that the convertible helicopter is potentially capable of establishing itself in a certain sector of the air transport field, *provided that the technical problems of conversion can be solved in a simple and practical manner*. This is the premise for the inclusion of a convertible helicopter in the comparative analysis in Section 5, which is mainly concerned with the question of the practicable worth of such aircraft. Another rather tentative conclusion from the foregoing survey concerns the optimum type of convertible helicopter. The author believes that one of the most promising forms (Fig. 51) comprises a more or less

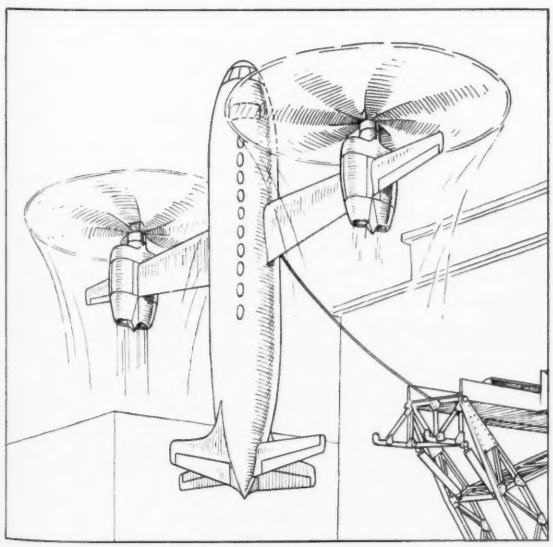


FIGURE 52. Convertible helicopter hovering near gantry with cable.

standard aeroplane body, a pair of relatively small fixed wings designed to give optimum performance at cruising flight, engines mounted near the wing tips and driving two rotors but, as well, interconnected to form *one* transmission system as in conventional helicopters. The rotors sustain the weight of the aircraft in the hovering condition of flight when the longitudinal axis of the aircraft is vertical, and act as propellers in the cruising condition of flight when this axis is horizontal; thus the aircraft as a whole, rather than one part of it, is rotated.

The conversion from one extreme flight condition to the other is slow or quasi-static. Thus when converting from the hovering condition the aircraft axis is at first slightly inclined in the normal plane of the aircraft, which causes a horizontal component of the rotor thrust. This gradually accelerates the aircraft horizontally, and the attainment of a small horizontal velocity, together with the induced velocity, results in an air flow which produces a small normal force on the wing in a direction opposite to the motion, that is to say, a small vertical component is introduced. A further tilt of the aircraft increases the horizontal velocity and causes a greater force on the wing so increasing the vertical component. This force grows until the reaction on the wing is vertical and equal to the aircraft weight. The aircraft will then fly at high speed and with its axis in the direction of flight. The conversion back to helicopter flight is the reverse of this process.

It can be shown that a high induced velocity in the hovering condition is necessary to avoid stalling of the wing during the conversion, *i.e.* the disc-loading must be high and the rotor diameters are consequently small. For this reason the power required curve of this convertible is much higher at the hovering point A than that of the conventional helicopter, whereas elsewhere both curves are similar. In order to meet this power characteristic, the power installation of such a convertible is preferably comprised of free-turbine power units—one pair on each side and of such power that, if necessary, any three units can sustain the aircraft during hovering (for safety). For normal cruising flight only two engines need be used (at their optimum cruising power), which naturally ensures a low fuel consumption. The aircraft is provided with two pockets in its wings fitted approximately in the position usually occupied by the conventional undercarriage. These pockets represent strong points capable of carrying the weight of the aircraft. Rotor control is used both during hovering and cruising flight, but is supplemented by certain fixed-wing controls. In cruising flight the pilots lie in a prone position, whereas during hovering they stand upright, the cockpit (situated in the nose of the aircraft) being in this condition somewhat akin to a circular bridge of a ship. The passengers are seated in rotatable seats which automatically maintain a level attitude during the process of conversion (Fig. 54).

Landing (Fig. 52) is made by means of a specially designed gantry in the following manner:

The landing gantry comprises fittings, not unlike the terminal buffers in railway stations, which contain shock absorbers, that can yield in all directions and which fit the pockets in the wings of the aircraft. The helicopter

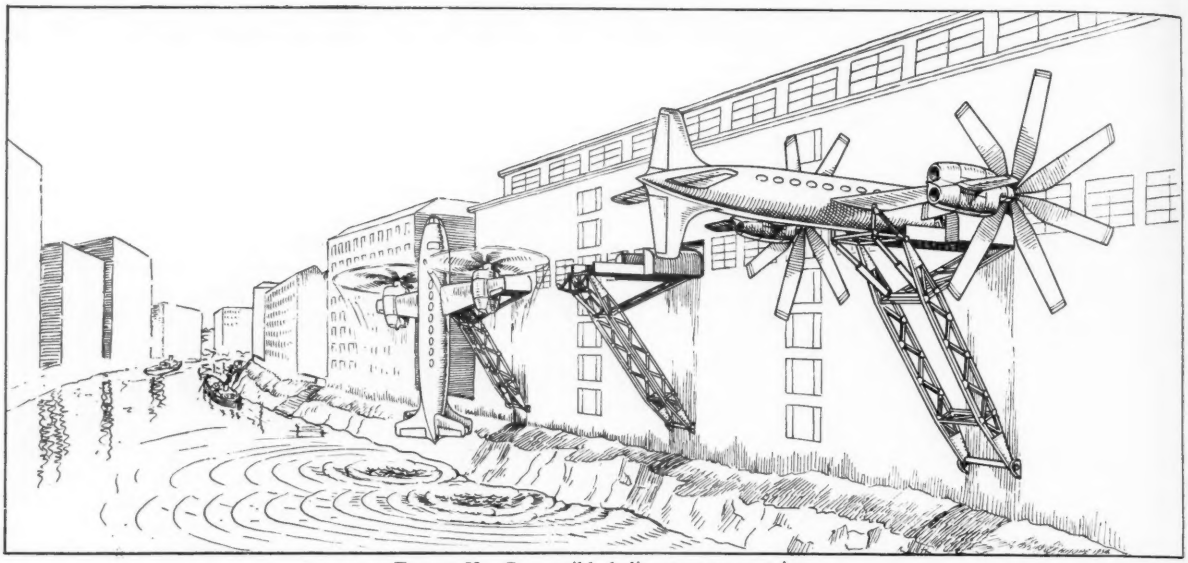


FIGURE 53. Convertible helicopters on gantries.

approaches the gantry in the hovering attitude until it comes to within about 50 yards of it. A steel cable is then lowered from a specially designed power winch, fitted at the c.g. of the aircraft, and which is under the control of the pilot. The end of the steel cable is then attached to an anchor point on the gantry. The pilot thereafter continues to hover the aircraft which is then winched in towards the gantry until the spring buffers register in the pockets of the aircraft.

The aircraft having been safely secured to the gantry, its engines are stopped and it is then rotated by a small ram into the horizontal position, after which unloading can begin (Fig. 53). A converse procedure applies to the take-off. The pockets on the aircraft and the landing buffers of the gantry should be standardised—like the gauge of the railways—to permit interchangeability of various makes of helicopters. A number of such gantries are provided in a rotor station, permitting the simultaneous operation of a number of helicopters.

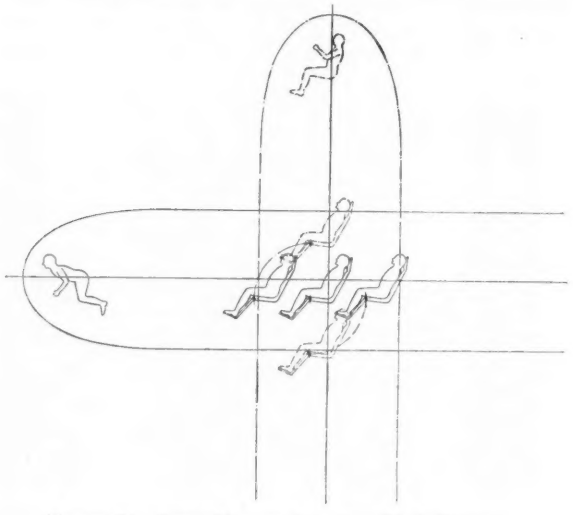


FIGURE 54. Rotatable seats in convertible helicopter.

In the event of two or more engines failing together, an emergency landing in aeroplane fashion can be made with the convertible on an ordinary runway, in which case the rotors suffer severe damage.

There remains the question of comfort. The rotation of the aircraft during take-off and landing may be considered an unpleasant or even frightening experience for passengers. It should be borne in mind that there is nothing physically unpleasant in this experience. Linear and angular accelerations are almost nil and certainly less than those in a conventional take-off. The experience is in fact only visual. The author believes therefore that there can be only psychological reasons for discomfort, and these will disappear with increasing familiarity of a new mode of transport. The public, after all, took reasonably quickly to the transport aeroplane and this, coming from railway trains and ships, was a larger step than a small difference in the method of take-off. In other points the comfort of the convertible is good. There is virtually no vibration in forward flight and in hovering only during conversion. This aircraft with its large low-speed rotors (or propellers) and turbines is fundamentally less noisy than any other known aircraft, which will be appreciated by those who have heard the Bristol Britannia. A low noise level is of the greatest importance for aircraft operation in urban areas.

5. The Helicopter in the Civil Air Transport Field and the Economics of its Operation Compared With Those of the Aeroplane

5.1. GENERAL

The ever-quickening expansion of aviation is exemplified not only by the growth in numbers and sizes, but also in the development of variety. The period from

1903 to about 1930 was the absolute realm of the piston-engined propeller-driven classic aeroplane. Then came in rapid succession the helicopter, the jet engine, the conquest of the sound "barrier," supersonic flight, and now today the acute problem of the heat "barrier" is being discussed.

That the aeronautical sciences have grown into a thousand and one departments each developing in its own direction is self-evident, and the interested observer or potential user of aeronautical facilities may well stand somewhat bewildered before this diversity and ask the question: where does it all lead? Is it a move in a particular direction or is it a spread in all directions? Is there a common denominator and can one make comparisons or is the whole fabric a tower of Babel?

This section is concerned with these questions in so far as they apply to the civil air transport field. An attempt is made here to relate a number of typical examples of air transport (some of which seem odd bedfellows in this wide compass of aeronautics) and for this purpose economics have been chosen as the yardstick: it is, after all, the rules of economics which, in the long run, ordain the balance in the diverse activities of our society. It is, then, the object of these considerations to ascertain what forms of air transport are likely to develop and precisely what their respective spheres of profitable service may be. There are without doubt many difficulties in judging the infinite varieties between chalk and cheese, and some of these will be elaborated in the following examples.

There is for example the question of aircraft size. An air service which connects a large number of small communities separated by short distances will meet the public demand best with relatively small transport aircraft operating at high frequency, whereas for a service between large centres, farther apart, the larger aircraft is indicated. Likewise a particular type of aircraft may be more suited for large size construction, whereas another may be better for large scale production. Allowance, therefore, must be made for such differences, but if the factors collated differ in almost every respect, then comparison becomes meaningless. The most frequently valuable comparisons are usually those which are specific.

If a family of abstract aircraft and their operation is chosen for comparison, then it may be argued that many imponderable but important details of design are left out and this can give misleading results; on the other hand if actual aircraft are used as examples, some factors which are irrelevant to the comparison are inevitably included and this also is misleading.

Certain compromises are therefore necessary in this investigation, and the plan which has been adopted is outlined in the following paragraphs.

5.2. ASSUMPTIONS AND METHOD OF COMPARISON

Eight types of aircraft have been selected for comparison, such that between them they cover at least a major part of the civil air transport field.

In the order of their cruising speeds they are as shown in Table I.

The standard of design and production is assumed to be the same for all types. Thus components which give the same performance or serve the same purpose equally well weigh the same, consume the same and cost the same. In particular the passenger unit, consisting of the passenger, his luggage, his seat and a share of the furnishing and the air conditioning, is assumed to be the same in all types considered with the exception of the supersonic aircraft, in which a small allowance is made for cabin cooling. The scale of production is assumed to be 100 units. The performance estimates for the helicopters are in agreement with the formulae given in Section 2 of this paper while those for the aeroplane are derived by standard performance methods. Annual utilisation is assumed to be 2,000 hours and the load factor is taken to be 75 per cent. All aircraft are capable of safe completion of the flight in the event of one engine failing at any time during flight. The rate of insurance (assuming 2,000 hours annual utilisation) plus interest on capital, is 9 per cent. per annum. All aircraft are assumed to have an economic life of 8 years. The price of fuel (gas turbine oil in all cases) is £0·1 per gallon. Flying crew costs are assumed to be £6 per hour. The allowance made for average head wind, is 30 knots (34·55 miles/hr.) for the aeroplanes and the convertible helicopter, and 4 knots (4·61 miles/hr.) for the other helicopters because the latter fly at lower height. The assumptions for maintenance and spares are the same in all cases.

TABLE I

(1)	A pure helicopter, designed for best economy at low cruising speed.
(2)	A pure helicopter, designed for best economy at moderate cruising speed.
(3)	A compound helicopter, designed for the same cruising speed as for (2).
(4)	A compound helicopter, designed for higher cruising speed.
(5)	A convertible helicopter, designed for best economy at high cruising speed.
(6)	A propeller-driven aeroplane, designed for the same cruising speed as for (5).
(7)	A jet-driven aeroplane, designed to cruise at high subsonic speed.
(8)	A jet-driven aeroplane, designed to cruise at supersonic speed.

The time spent by the passenger for the total journey from town terminal to town terminal is considered to be a financial loss to him and is charged at a rate of £1 per hour[26].

The optimum weights of these aircraft spread over a wide range and would follow approximately the order given in Table I. At the one extreme, a pure helicopter, suited for operation over ultra-short stage lengths, would

TABLE II

	Item	*Rate/lb.*	*Weight*	*Cost of Production*
(1)	Airframe equipped ...	r_1	w_1	r_1 w_1
(2)	Power-plant complete ...	r_2	w_2	r_2 w_2
(3)	Propeller or rotor hubs, gearboxes and shaftings	r_3	w_3	r_3 w_3
(4)	Propeller or rotor blades	r_4	w_4	r_4 w_4
	Total	Average cost rate for whole aircraft $=\Sigma (r_n w_n)/\Sigma (w_n)$	$\Sigma (w_n)$ empty weight	$\Sigma (r_n w_n)$

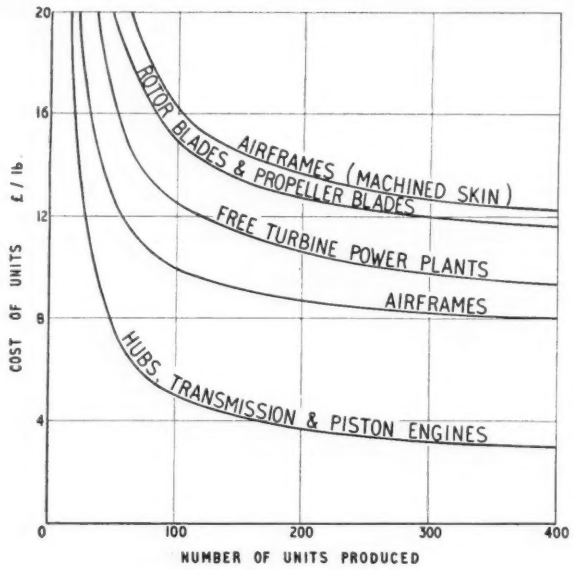

COST OF UNITS £ / lb
NUMBER OF UNITS PRODUCED
ROTOR BLADES & PROPELLER BLADES
AIRFRAMES (MACHINED SKIN)
FREE TURBINE POWER PLANTS
AIRFRAMES
HUBS, TRANSMISSION & PISTON ENGINES

FIGURE 55. Typical supply curves.

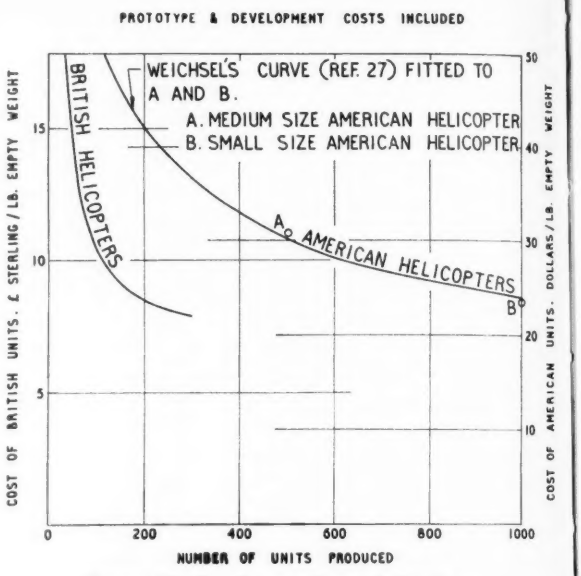

PROTOTYPE & DEVELOPMENT COSTS INCLUDED
WEICHSEL'S CURVE (REF. 27) FITTED TO A AND B.
A. MEDIUM SIZE AMERICAN HELICOPTER
B. SMALL SIZE AMERICAN HELICOPTER
BRITISH HELICOPTERS
AMERICAN HELICOPTERS
COST OF BRITISH UNITS. £ STERLING / LB. EMPTY WEIGHT
COST OF AMERICAN UNITS. DOLLARS / LB. EMPTY WEIGHT
NUMBER OF UNITS PRODUCED

FIGURE 56. Supply curve for typical helicopter.

have an all-up weight of about 30,000 lb., intermediately a propeller or jet-driven aeroplane for 2,000 miles stage length would gross about 100,000 lb. and at the other extreme a supersonic long-range aeroplane appears to be still too small at 200,000 lb. A.U.W.

However, it has been found most desirable to fix the all-up weight as a common factor in this family of aircraft, for by this means most of the significant differences, which otherwise would have been hidden, appear explicitly in the analysis. The compromise made is 50,000 lb. A.U.W. for all aircraft.

Some difficulty is also experienced in the assessment of the first cost of aircraft. Table II appears to be a flexible and yet simple method: the empty weight of the aircraft is divided into lots, which come under four basic headings representing different kinds of engineering.

Each category of engineering has its own supply curve defined by the cost of production per pound weight (or cost rate) against the scale of production. Because of the need for tooling and jigging the cost rate is very high for small production numbers, and falls as the production increases. This feature is most pronounced in machined parts (where there is a relatively high content for tooling in the cost rate) which are suited for mass production. Fig. 55 gives typical supply curves based on experience in the United Kingdom. This may be compared with American figures. (The latter relate to the whole aircraft, *i.e.* to the expression $\frac{\Sigma (r_n w_n)}{\Sigma (w_n)}$ in Table II.)

TABLE III

Number of units produced	*Average unit cost*
50	1·38
100	1·00
500	0·50
1000	0·38

A small helicopter with a production of over 1,000 —$23.00/lb.

A larger helicopter with smaller production —$30.00-$35.00

Also for comparison with the shape of the supply curves in Fig. 51, the relative figures, quoted from an American source[27], are given in Table III.

A supply curve for a whole helicopter has been prepared (Fig. 56) from the information in Fig. 55 and assuming a weight breakdown of a typical helicopter. The first costs must include a share of the prototype and development costs. Typical British figures for these items are £40 (per lb. empty weight per prototype) for aeroplanes and £60 for helicopters. Corresponding American figures are:

$2,000/lb. for a small helicopter
$1,160/lb. for a medium helicopter
$225/lb. for a large helicopter.

These latter figures are interesting, as they show a reduction with increase of helicopter weight. If this is not incidental it could be explained by the fact that a portion of the cost of development work is independent of the size of the aircraft.

The average is for two to four prototype aircraft to be built and their cost has to be discharged during the subsequent production. Thus the share of development expenses which each production aircraft carries (by the strict economic argument) is cost of prototypes/scale of production. Production cost and profits, together with the latter item, thus give the first costs.

It is assumed that the aeroplanes operate between airports, which are connected by special 'bus services with the towns they serve. The airline operator maintains offices at these airports as well as in the centres of towns. The conventional form of airline handling of passengers and their luggage is assumed. The airport landing charge for a 50,000 lb. aeroplane is taken to be

£12·5. This figure is based on the assumption that the airport is maintained without subsidy, which does not generally apply today. The time allowance for the journeys between airport and town (measured from the times of take-off and landing) is calculated as 1·75 hour: this does not allow for passport and similar formalities. A charge of £0·25 is assumed for the journey between airport and town centre.

The helicopters are assumed to operate between rotor stations which are situated in the centres of the towns. These rotor stations could well be the roofs of large buildings providing ample office and other accommodation, not only for the airline operator, but to an even larger extent for other occupiers. The technical equipment on these rotor stations is assumed to be similar to those on airports (airport control, meteorological service, and so on).

The transit of passengers and their luggage through the rotor stations is assumed to be similar to that on railway stations. There is normally no advance booking and tickets are bought at the start of the journey. This arrangement is not inconvenient for passengers if the service operates at reasonable frequency from centrally situated points, so that in the event of a full aircraft, the next can be waited for or other dispositions can readily be made. The terminal time losses for passengers, that is the time from the instant of entering the rotor station to that of take-off, plus the time from the point of landing to that of leaving the building, is taken to be 0·1 hours. It does not allow for customs and similar formalities.

The landing charge for the services on the rotor station is estimated as £6·25. This is lower than the airport charge because of the more economical use of ground and equipment. The area of the rotor station is only 1/1,300 of that of the aerodrome and, moreover, it uses the roof portion of the building, which would otherwise be only an item of expenditure (Fig. 57)[31]. The part connected with helicopter operation constitutes only about 10 per cent. of the total value of the building, the remainder serving other purposes, whereas in an airport practically the whole revenue must come from the flying operations.

Both the helicopter and the aeroplane have facilities for diversion but it is assumed that for the helicopter the distances involved are less. The convertible helicopter, in particular, can have ample provision of landing gantries. It is therefore justifiable to assume smaller fuel reserves for the helicopter:

FUEL RESERVES

Supersonic Aeroplane—0 miles for diversion plus ¼ hour holding.

Subsonic Aeroplane—300 miles for diversion plus ¾ hour holding.

Helicopter—20 miles for diversion plus ¾ hour holding.

The costs of the eight different operations have been computed under two headings:

(*a*) direct operating costs of the aircraft

(*b*) the airline costs including overheads.

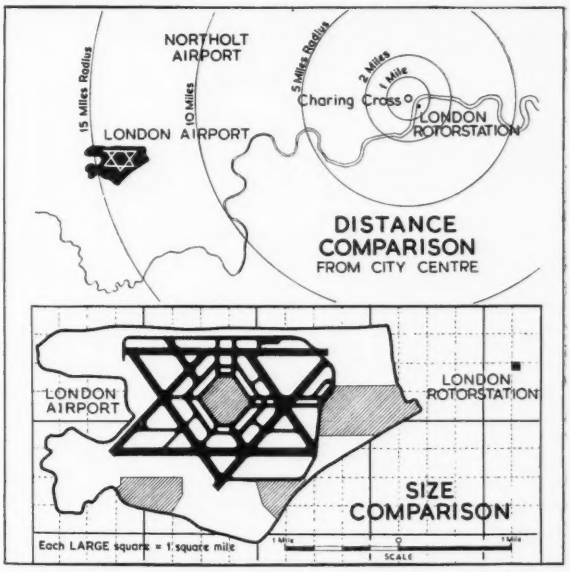


FIGURE 57. Rotorstation v. airport. Geographical advantages.

The direct operating costs, with minor exceptions, are based on a method of computation proposed by British European Airways[28], which is very similar to that of the Society of British Aircraft Constructors[29]. It is given in Appendix III and needs no further comment here.

Appendix IV shows a weight and first-cost analysis based on the various arguments which have been outlined earlier in the paper. From these and other data the direct operating costs have been obtained. They have been related to the transport performance of the aircraft, which produces direct operating costs per seat nautical-mile as a function of stage length (Fig. 58).

Typical airline costs have been computed from information supplied by British European Airways[30] on the same basis as the direct operating costs, that is under headings of annual costs, hourly costs and costs connected with landings. This information is directly applicable to aeroplane operation, but does not reflect the contracted and simplified organisation possible with the operation from rotor stations and which materially reduces the overhead and similar costs. These reductions are shown in Table IV which gives the airline costs for both types of aircraft.

From the figures shown in Table IV (as with the direct operating costs) the airline costs per seat nautical-mile have been obtained as a function of stage length (Fig. 59). These considerations also produce the total journey time (inclusive of terminal time losses) as a function of stage length, and this is shown in Fig. 60.

TABLE IV

AIRLINE COSTS FOR A 50,000 LB. AIRCRAFT

Aircraft	*Annual*	*Hourly*	*Per landing*
Helicopter	£40,000	£6·7	£46
Aeroplane	£45,000	£6·7	£68

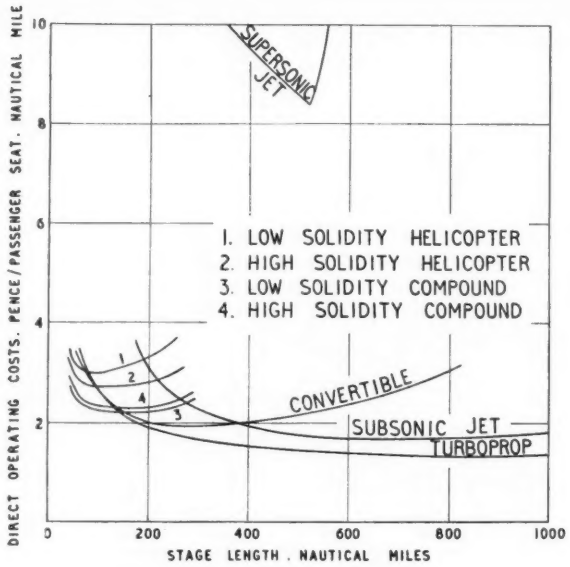


FIGURE 58. Direct operating costs.

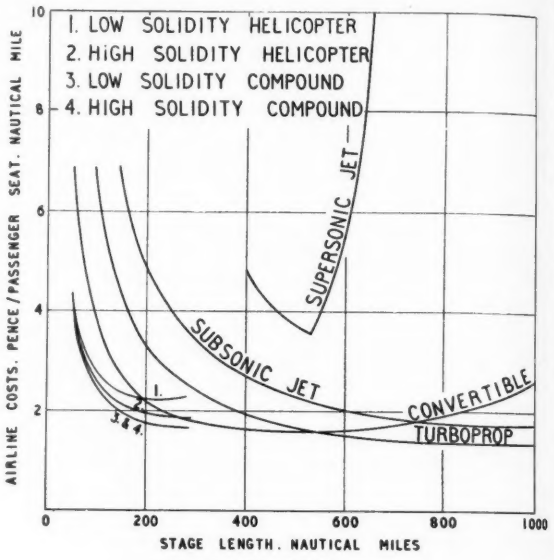


FIGURE 59. Airline costs.

A final operation to show comparative journey costs was made by adding the values of Figs. 58 and 59, allowing for the load factor, multiplying by the stage length and adding the money equivalent of the journey time shown in Fig. 60 together with the 'bus fare, where applicable. The comparative journey costs are given in Fig. 61.

5.3. DISCUSSION OF RESULTS

It will be seen that in terms of direct operating costs (Fig. 58) the subsonic jet and the turbo-prop aeroplane are without competition on long journeys. They are both, however, and especially the jet, less efficient in short-haul work because of the reduced block speeds in this regime of operation.

But here the helicopter makes its claim. The superiority of the compound helicopter over the pure helicopter will be noted, this being mainly due to the higher speed of the compound. Another indication is the effect of rotor solidity. The substantial increase in the cruising speed of the slow pure helicopter, obtained through increase of solidity, represents a large and economically favourable factor, which more than balances the increased rotor profile power. But the same increase in solidity in the faster compound helicopter does not produce a sufficient speed increase to outweigh increased rotor profile power and similar adverse factors at this higher speed. Moreover, it will be seen that the high solidity pure helicopter is 0·6 pence

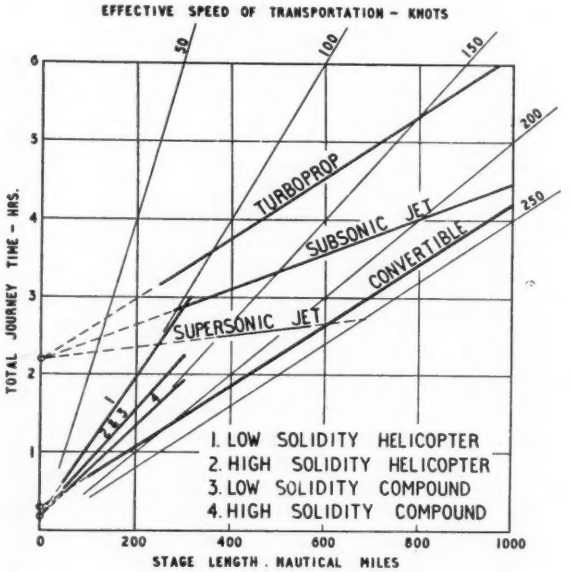


FIGURE 60. Total journey time.

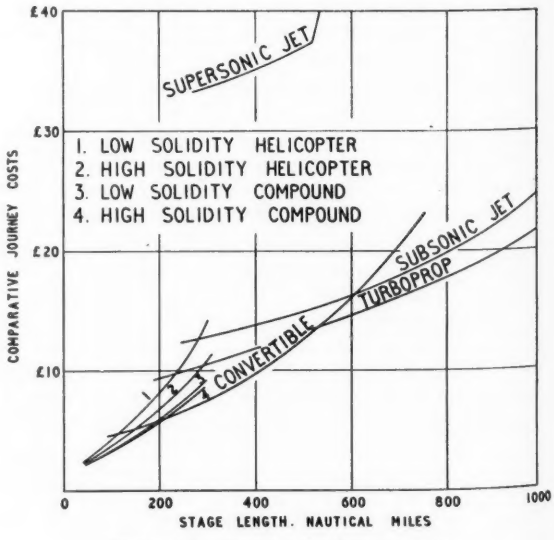


FIGURE 61. Comparative journey costs.

per seat nautical-miles more expensive to operate than the low solidity compound helicopter of the same cruising speed. It is evident that area vested in the fixed wing is better than area in the rotating wing. The convertible helicopter lies halfway between the other helicopters and the subsonic aeroplanes.

The supersonic aeroplane, cruising at about $M=2$, is quite uneconomic. This aircraft has rather suffered in this comparison because it is so much removed from its optimum size. There are indications that with increase in A.U.W. its curve will move downward and slightly to the right, but even then it does not appear to be an economic proposition. Really high speeds can probably be achieved economically only outside the earth's atmospheric envelope, and the true rocket vehicle, therefore, may be the most practicable form of high speed aircraft.

The airline costs (Fig. 59) present a similar picture as the argument in this case is much the same. The curves for the different operations, however, appear to lie a little closer to one another.

A most revealing graph is that showing total journey time (Fig. 60). The combined terminal time losses (in flying and on the ground) are shown vertically on the left hand side of the graph, where the lines begin. The slope of the lines indicates cruising speed, the steeper the line the slower the aircraft. It is evident that, for short-haul journeys the slow helicopters are much faster than the fast aeroplanes. Indeed the pure and compound helicopters have run out of range and come to the end of their longest journeys, almost before the aeroplanes have been able to start their flights. A typical helicopter layout for this type of work is shown in Fig. 62.

The turbo-prop and the subsonic jet aeroplane are in their element for journeys from 1,000 miles upwards.

The supersonic aeroplane reaches its best effective speed of transport at a range of about 600 miles when, however, it runs out of fuel; this could be delayed with a larger machine. An outline of the "small" 50,000 lb. supersonic aeroplane considered in this comparison is given in Fig. 63. Here 8 to 12 people sit in a single row in a narrow cabin without corridor, for a duration of not more than half an hour. This time is very short compared with that spent on the ground.

The most attractive line in Fig. 60 is that of the convertible helicopter, which appears to be potentially capable of combining the advantages of the helicopter with those of the aeroplane. It will be noted that the convertible overtakes the other helicopters in a very short distance. It could be caught by the supersonic aeroplane at 600 miles out, provided that the latter had enough fuel; the subsonic jet aeroplane reaches it at about 1,200 miles, and the turbo-prop remains always about two hours behind it.

The final criteria are the comparative journey costs of the various forms of transport (Fig. 61). It is evident that the supersonic aeroplane is not competitive at *any* stage length. The subsonic jet aeroplane cannot compete with the turbo-prop aeroplane, the difference between the two being about £2 throughout almost the

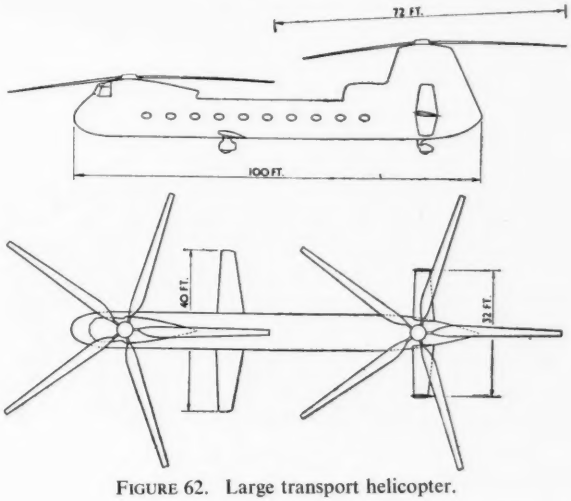


FIGURE 62. Large transport helicopter.

whole range shown in the graph. The compound helicopter is notably better than the pure helicopter.

The convertible helicopter has no rival between 200 and 530 miles stage lengths.

There remains a factor which even though it is economically imponderable, has significant economic effects. It is the convenience of the service, the comfort of the customer. Where comfort is lacking demand falls quickly. One of the most irritating experiences in conventional air journeys is the changing from coach to aircraft and then back to another coach. The arrival, instead of being the climax of a journey, very often becomes an anti-climax. A jolting 'bus which, between starts and stops on dreary suburban roads, approaches the final destination in a seemingly uncertain manner could scarcely be more calculated to make everybody feel gratification that: "at last this is over."

There is no doubt that avoiding changing on a journey greatly enhances the comfort, apart from reducing the risk of lost luggage or similar mishaps. It is difficult generally to assess the value of such comfort, but the author personally would be prepared to pay the 'bus fare for the privilege of *not* having to use the 'bus.

This is a powerful factor in favour of the helicopter which could not be included in the foregoing comparison.

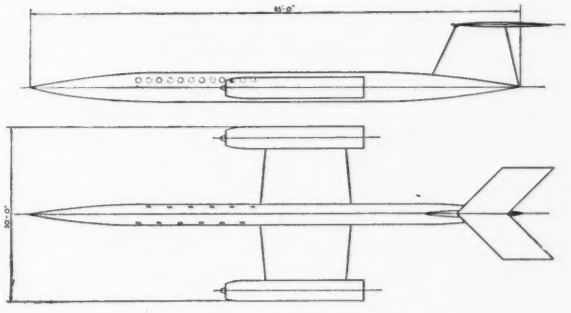


FIGURE 63. Supersonic aircraft.

FIGURE 64. Bristol Sycamore—Royal Australian Navy.

6. The Uses of the Helicopter in the National Services and other Specialised Uses

6.1. GENERAL

Apart from the general field of commercial air transport, there is a wide use for the helicopter in a range of specific roles, the most important of which are those concerned in the national defence. Typical of such specific applications are anti-submarine warfare, casualty evacuation, pest control, power-line inspection, air charter work, and geophysical exploration, but there are many more.

In all such special roles the helicopter must satisfy economic criteria. However, they are no longer expressed by the operating cost per seat mile and the simple rule of a passenger's time being worth one pound sterling per hour; rather different economic arguments apply which, indeed, vary from one role to another.

Nearest in character to the general transport helicopter come the helicopter taxi and the private helicopter of the top executive. The time-saving in these cases is obtained in the same way as in the general transport operation and, although the time saved is generally not very much more, it is worth while because the price which is paid for time is higher.

Other roles belonging to this category are the surveying of ground for prospecting and similar purposes, and the inspection of power lines and pipe lines. Here the price which is paid for time (of the operators) will probably not be greater than that in the general transport role, but the saving of time arises from different considerations. In these roles the helicopter is certainly not faster, in fact it may be considerably slower than in the general transport role, but the operation takes place in such geographical circumstances that the alternative to the helicopter would involve slow movement on the ground, interspersed by frequent stops. Here then the helicopter saves a major aggregate of operators' time.

Another role is connected with the transport of goods. This is appropriate only in places where there is no other reasonable form of transport. In such operations the question of actual journey time does not arise as an important issue, but the relatively high price for transportation by helicopter is justified because any alternative would involve either extremely inefficient transport, or a major capital outlay for the building of a road or railway. A typical example for this kind of work is the construction of a hydro-electric power station in British Columbia, in which helicopters play an important part (Fig. 65).

A different economic argument applies to the fire-fighting services, especially those in inaccessible areas. In this case time is the all-important factor, the value of a given period of time being exactly equal to the material value destroyed by the fire during that same length of time. A forest fire, for example, after a very brief acceleration period, reaches a high rate of burning and passes eventually beyond control. In this case the worth of time is consequently not linear; the first minutes are the most precious, after which the value of time deteriorates until finally it reaches nil when all is lost and, therefore, nothing is left to save. Thus the price justified for a quick and effective means of fire fighting is rather higher than for a somewhat inferior method, but the difference in value between a slow and a very slow emergency service is probably next to nothing. The same applies to flood control.

This argument assumes different proportions for direct rescue operations. Here, whether it is in bringing medical assistance to the rescuee or taking him to a place where such assistance can be given, the time factor is even more critical, and the value at stake is human life. Clearly there can be no question of compromise, the fastest service should be provided irrespective of the expense. Sometimes circumstances arise in which certain supplies or services occupy a position of monopoly. The price is then determined by the supplier, and whether this price is paid depends on the desire or necessity of the user. When, however, such monopoly

FIGURE 65. Bell 47 helicopter in Canada.

enters the sphere of life-saving operations, as for instance during the floods in Holland early in 1953, where in certain places the only chance of survival was through rescue by helicopter, then such service becomes literally priceless.

This argument reaches the ultimate measure when the life of the nation is at stake, as in war. In the emergency of war the stakes are so high that no effort is too great, no price can be too high, to save the country. Only the best is good enough, whatever its cost may be.

The value of the helicopter in all special roles does not originate from its cruising speed, which is moderate, but from its ability to move in a confined space, to hover, and to land on an area not much greater than its undercarriage. It is therefore eminently suited for improvisation, which gives it mobility, ubiquity and adaptability in emergencies.

A number of such special roles, both in the Services and the civilian sphere, are enumerated.

6.2. NAVAL USES

One of the greatest threats to the ocean routes on which the life-blood of the nation depends is the submarine; a formidable weapon, difficult to detect, to hunt and to kill. In the last war its destruction was the task of the destroyer and kindred ships and the carrier-borne or land-based aeroplane. These operations are elaborate and expensive. The helicopter appears to be more suited for this task as it has a sufficient speed margin over the submarine to make quick contact and combined with this, the ability to move slowly or to hover for accurate location of the target and eventual attack. Moreover, the helicopter can operate from any ship, obviating the need for the expensive combination of carrier and aeroplane. Another obvious duty of the helicopter is minesweeping, as it is relatively immune from the effects of mine detonation. For communication duties it can carry anything from a message to the admiral of the fleet himself. In the air/sea rescue role the helicopter has already established itself as a necessity for aircraft-carrier operations, and it has also been used for aeroplane-guard duties with carriers, so freeing for other work the destroyers otherwise used (Fig. 64).

6.3. USES IN THE ARMY

Where ground communications are restricted the helicopter is the ideal transport vehicle for both troops and cargo. It is particularly useful in the bridging of obstacles like rivers and minefields. The same applies to the communications role, *i.e.* the transport of persons, messages or the laying of telephone lines. These features greatly increase the tactical mobility of battle units, which is such an important factor in modern land warfare[33].

Another important role is in the medical service—particularly the evacuation of casualties from forward battle areas—and this is a major factor in the morale of the men.

It is, in addition, considered that the helicopter is capable of accomplishing with superior efficiency almost all duties performed by road vehicles.

6.4. SPECIAL CIVIL ROLES

These roles are widely diverse and one can do little more here than cite a few representative examples. In the national service, then, the helicopter could effectively combine many of the coastguard duties, the life boat service, forest patrol and fire fighting, all forms of rescue work as well as police duties. Helicopters especially equipped for these combined operations and suitably distributed throughout the country would offer a maximum of service to the community for a given expenditure.

As soon as more landing facilities are available for the helicopter, its potential use will be increased considerably and the helicopter taxi and the private helicopter of the top executive may then become a familiar sight. Other already well established functions of the helicopter include the following: pest control, power-line inspection, geophysic exploration, construction work, whale hunting, harbour work, petroleum and other prospecting.

7. Concluding Remarks

7.1. SUMMING UP AND LOOKING FORWARD

Reviewing briefly the ground covered in the several sections of the paper a number of salient features are seen to be noteworthy.

In the historic study it is evident that to the early investigators the rotating wing appeared to offer better promise in the long run than did the fixed wing. The relative simplicity of the latter, however, brought the first practical results so leading it to success but, at the same time, unfortunately, retarding the progress of other possibilities. It was only some time later that the rotating wing again engaged the general aeronautical interest, mainly because the short-comings of the fixed wing had begun to assume threatening proportions. Today the rotating wing is well established in many specialised roles and it is also making a footing in the short-haul transport role.

The high speed limitation of the rotor and, therefore, its range of action, is much lower than that of the fixed wing; on the other hand the *low speed limit* of the rotor is *zero*. This fact is the cue to the usefulness of the helicopter. The rotating wing requires only a simple ground organisation for its operation, and it can link directly the centres of cities.

A form of convertible helicopter is described which appears to have attractive possibilities. The warning is given, however, that a number of major aerodynamic, technical and navigational problems require solution before such an aircraft can become a reality.

In the economic analysis it is shown that both the pure and the compound helicopter can hold their own in the ultra-short-haul field of operation, namely between stage lengths of 25 and 200 miles. From 200 miles up to 530 miles stage length, the convertible helicopter has the obvious advantage. For stages above 530 miles the aeroplane is supreme as long as there is air and fuel. A significant indication in this analysis is in the airline costs of the various types of aircraft. These are considerably reduced in the simplified operations from rotor stations. A paradoxical conclusion is that in

short-haul work, the slowly cruising helicopter has a much higher effective speed of transport than has the fast aeroplane. In particular, the convertible helicopter is faster than an aeroplane flying at a speed of Mach 2·0! The supersonic aircraft does not appear to have an economic justification at *any* stage length.

Much of the aeronautical world is now in the melting pot and when the new shapes emerge it may well be found that the picture has changed somewhat:

(1) Various forms of helicopter may handle the vast volume of traffic up to journeys of 500 miles.

(2) The trans-continental and trans-ocean traffic may be undertaken with the high-speed *subsonic* airliner.

(3) For military high-speed flying and for special roles, where there are entirely different economic criteria, the supersonic aircraft may be used, especially in the form of the guided missile.

But supersonic flight does not appear to be *economically* tenable in commercial terms, and high speeds will probably be achieved only outside the earth's atmosphere. Thus looking into the farther future, we may find the space rocket taking travellers from Europe to America, and a vehicle in a vacuum tube bringing shoppers from London to Paris; and this—alas—may well be the swan song of aeronautics.

Until then, however, a reasonable span of time may be expected during which we may apply some of our thoughts and energies to the helicopter.

The rotating wing is revolutionary in the fullest sense of the word; it has outmoded much past thinking, and it demands an uninhibited and imaginative approach.

Let our imagination, therefore, escape the shackles of convention and win that freedom that the rotating wing enjoys. We may then apply fittingly, both to our thoughts and our efforts, the precept:

"En plein vol . . . plus haut, toujours plus haut—plus loin, plus loin toujours!"[32].

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APPENDIX I.

ATTEMPTS TO FLY PRIOR TO 1903.

Type: O—Ornithopter. H—Helicopter. A—Aeroplane.

Name	*Type*	*Date*
Daedalus and Icarus		
King Bladud		C.852 B.C.
Archytas of Tarentum		5th Cent. B.C.
Simon the Magician		C.67 A.D.
Wayland the Scandinavian		
Oliver of Malmesbury		C.1060
The Saracen at Constantinople		C.1178
Jean-Baptiste de Pérouse		1420
J. B. Danti		1490
Leonardo da Vinci	O	C.1500
Leonardo da Vinci	H	C.1500
John Damian, Abbot of Tungland		1507
P. Guidotti		1569
R. Hooke		1655
Allard	O	C.1660
Bernain	O	1673
Besnier	O	1678
De Bacqueville	O	1742
Childs	O	1757
A. J. P. Paucton	H	1768
Abbé Desforges	O	1772
C. F. Meerwein		1781
J. P. Blanchard	O	1781
J. P. Blanchard	H	1784
Gerard	O	1784
Launoy and Bienvenu	H	1784
Sir G. Cayley	H	1795
Sir G. Cayley	O	1808
Sir G. Cayley	H	1843
J. Degen	O	1809
J. Degen	H	1816
Count Lambertye	H	1818
Sarti	H	1823
Meyer	H	1828
Dubochet	H	1834
Cagnard	H	1835
W. H. Phillips	H	1842
Bourne	H	1843
Miller	O	1843
Cossus	H	1845
Henson	A	1842
Séguin	H	1846
Stringfellow	A	1848
Stringfellow	A	1868
Renoir	H	1848
Aubaud	H/A	1851
J. M. Le Bris	H	1851
J. M. Le Bris	O	1857
J. M. Le Bris	A	1867
Letur	O	1852
Loup	A	1852
Breant	O	1854
Carlingford	A	1856
F. du Temple	A	1857
F. du Temple	A	1874
Jullien	A	1858
H. Bright	H	1859
Smythies	A	1860
Franciso Orujo	A	1863
De Louvrié	A	1863
De Louvrié	O	1877
Nadar	H	1863
Claudel	A	1864
De Groof	O	1864
D'Esterno	A	1864
Struvé and Telescheff	O	1864
Ponton d'Amécourt	H	1865
Mouillard	A	1865
Bourcart	O	1866
Smyth	A/H	1867
Butler and Edwards	A	1867
Brown	A	1868
Brown	A	1873
C. Spencer	O/A	1868
Gibson	O	1868
I. Palmer	O	1868
F. D. Artingstall	O	1868
I. M. Kaufmann	O	1868
A. Pénaud	H	1870
A. Pénaud	A	1871
A. Pénaud	O	1872
A. Pénaud	A	1876
M. G. Trouvé	O	1870
R. C. Jay	O	1871
R. C. Jay	O	1877
Prigent	O	1871
Joubert	O	1871
Joubert	O	1872
Danjard	A	1871
Pomès and De La Rue	H	1871
De Villéneuve	O	1872
C. Ader	O	1872
C. Ader	A	1886
C. Ader	A	1890
C. Ader	A	1897
Quinby	O	1872
Achenback	H	1874
T. Moy	A	1875
T. Moy	A	1879
Marriott	A	1875
Sénécal	A	1876
Ward	H	1876
V. Tatin	O	1876
V. Tatin	A	1879
F. X. Lamboley	O	1876
Dieuaide	H	1877
Mélikoff	H	1877
H. Murrell	O	1877
Barnett	A	1877
Castel	H	1878
Forlanini	H	1878
Pomès	A	1878
Linfield	A	1878

Name	*Type*	*Date*
Dandrieux	O	1879
F. W. Brearey	A	1879
F. W. Brearey	A	1885
Quinby	H	1879
Greenough	H	1879
T. A. Edison	O	1880
W. Beeson	A	1880
Krueger	A	1882
Goupil	A	1883
H. F. Phillips	A	1884
H. F. Phillips	A	1890
H. F. Phillips	A	1891
H. F. Phillips	A	1899
De Sanderval	A	1884
L. Hargrave (many machines)	A	1884 1903
Armour	A	1884
J. S. Foster	H	1885
M. C. Ring	A	1885
Renard	A	1889
Pichancourt	O	1889
W. Quatermain	O	1890
E. P. Frost	O	1890
H. Middleton	O	1890
De Graffigny	A	1890
O. Lilienthal	A	1891
O. Lilienthal	O	1895
O. Lilienthal	A	1896
Baden-Powell		1891 1903
S. P. Langley	A	1891
S. P. Langley	A	1896
S. P. Langley	A	1901
S. P. Langley	A	1903
Trouvé	A	1891
G. Koch	A	1891
W. G. Walker	H	1892
H. S. Maxim	A	1892
Huffaker	A	1893
J. J. Montgomery	A	1893
C. A. Parsons	H	1893
P. Pilcher	A	1895
P. Pilcher	A	1897
P. Pilcher	A	1899
Ludwig	H/O	1897
A. A. Merrill	A	1898
W. and O. Wright	A	1899
W. and O. Wright	A	1901
W. and O. Wright	A	1902
W. and O. Wright	A	1903
W. Kress	A	1901
Decazes and Besaçon	H/A	1902

APPENDIX II

HELICOPTERS AND GYROPLANES (1903 TO BEGINNING OF 1939-45 WAR)

Name	*Date*
Robert and Pillet	1904
C. Renard	1904
Deutsch de la Meurthe and Tatin	1904
A. and H. Dufaux	1905
A. and H. Dufaux	1908
Léger	1905
Santos-Dumont	1906
P. Cornu	1906
P. Cornu	1907
Bréguet and Richet	1907
Bréguet and Richet	1908
Julian Félipe	1907
O. Luyties	1907
H. Berliner	1908
H. Berliner	1920
H. Berliner	1922
Kimball	1908
Zanni	1908
J. Bertin and Boulline	1908
Monin	1908
Vuitton and Huber	1909
Denny and Mumford	1909
Denny and Mumford	1912
Denny and Mumford	1914
I. I. Sikorsky	1910
J. C. H. Ellehamer	1911
H. Villard	1913
Decazes	1913
Rougé	1914
Papin and Rouilly	1915
Crocker-Hewitt	1916
Petroczy and Von Kármán	1916
E. Douheret	1919
Perry	1919
R. P. Pescara	1919
R. P. Pescara	1923
R. P. Pescara	1925
R. S. Pescara	1925
Damblanc	1920
Oemichen	1920
Oemichen	1922
Oemichen	1928
G. de Bothezat	1922
J. de la Cierva	from 1923 to 1936
(Licencees—Wier, Kay, Westland, D.H., Avro, Comper, Parnall, Kellett, Pitcairn, Focke, L.e.O., S.N.C.A.S.E., etc.)	
Florine	1923
Florine	1933
L. Brennan	1923
Asboth	1924
Asboth	1925
Asboth	1927
Asboth	1928
Hellesen-Kahn	1925
Von Baumhauer	1925
L. Chauviere	1927
V. Isacco	1929
V. Isacco	?
Curtiss-Bleecker	1930
R. Hafner	from 1930 to date
D'Ascanio	1930
Bréguet-Dorand	from 1930 to date
G. Herrick	1931
Odier-Bessiere	1932
H. Focke	1932
H. Focke	1936
H. Focke	1938
H. Focke	1941
B. Nagler	1936
Yuriev	1932
I. Bratukhin	1934
A. Flettner	1938
A. Flettner	1940
J. Weir	1938
J. Weir	1939
I. I. Sikorsky	from 1939 to date

APPENDIX III

An extract from British European Airways "Outline Specification of Requirements for Large Helicopter"—October 1951, amended May 1952.

2. PERFORMANCE

The performance required under normal and one engine inoperative conditions is given in this section. Unless otherwise stated, maximum take-off weight and I.C.A.N. conditions are to be assumed. Heights are I.C.A.N. pressure heights.

Performance and weight estimates required from the designer are given in Appendices 3 and 4.

2.1. *Payload and Range.*

2.1.1. The payload shall be made up of passengers and baggage according to the following table:—

Service	*Approx. Range of Stage Lengths (n. miles)*	*Av. Pass. Weight (lb.)*	*Av. Baggage Wt./Pass. (lb.)*	*Total Pass. Unit (lb.)*
Internal	20-160	155	25	180
Continental	120-240	160	40	200

The payload for the average internal stage of 100 nautical miles shall be not less than 30 passengers, using the reserves and allowances of 2.1.3.

2.1.2. Accommodation shall be provided for the payload possible with a fuel load needed for the completion of a 20 nautical mile stage.

2.1.3. The *fuel load* for any stage is to be calculated on the engine manufacturers stated fuel consumption +3 per cent. at the power output required at 2,000 ft. for the average cruising speed defined in Para. 2.2.2 and for the following duration:

(*a*) the duration for the stage distance in a 30 knot headwind.

plus (*b*) 10 minutes take-off and landing allowance for time spent in ascent and descent without covering distance and for increased distance flown at terminals in conforming to traffic control requirements.

In addition the fuel required at the minimum power for level flight for:—

(*c*) 45 mins. hold-off duration.

2.1.4. *Tankage* shall be provided for a 200 nautical mile stage in a 40 knot headwind with the reserves and allowances. (*b*) and (*c*) of para. 2.1.3.

2.2. *Speed.*

2.2.1. The *design speed* at which the level of vibration shall not exceed that laid down in Para. 7.2 should be as high as possible; provided that in choosing the design (*e.g.* rotor parameters, power loading, parasite drag) to achieve a high design speed the basic aircraft type costs per ton-mile at the average cruising speed defined in Para. 2.2.2 and at a given annual utilisation do not significantly depart from the optimum. In any case, the design speed shall not be less than 130 knots. The manufacturer's design study should include a note on the reason for the choice of design parameters.

2.2.2 The *average cruising speed* to be used in the assessment of direct operating costs shall be not less than 120 knots or 90 per cent. of the design speed under the following conditions:—

(*a*) Power plants operating at maximum weak mixture power or 50 per cent. METO, whichever is the lesser, in the case of piston engines, or at 70 per cent. maximum power output in the case of turbine engines.

(*b*) The aircraft at 95 per cent. of the maximum take-off weight.

(*c*) Cruising height 2,000 ft.

2.2.3. The cruising speed at 5,000 ft. with the *critical engine inoperative* shall be not less than 100 knots at the maximum continuous power rating of the remaining engine(s).

2.3. *Climb.*

2.3.1. The vertical rate of climb at sea level and maximum power output shall be not less than 600 ft. per minute, without ground effect, and at zero forward airspeed.

2.3.2. The hovering ceiling without ground effect and with maximum power output shall be at least 5,000 ft.

2.3.3. With the *critical engine inoperative* and the remaining engine(s) operating at the maximum continuous power rating, the rate of climb at the best forward speed for climb shall be not less than 200 ft. per minute at all altitudes between sea level and 5,000 ft.

HELICOPTER DIRECT OPERATING COST (B.E.A. APPENDIX V)

ANNUAL COSTS — £/hr.

1. *Obsolescence* (Aircraft and Spares) — $0{\cdot}13\frac{C}{U}$
   (C = First cost of one equipped aircraft (£))
   (U = Utilisation per aircraft (hrs. p.a.))
2. Insurance (Aircraft and Third Party) — $0{\cdot}06\frac{C}{U}$
3. Interest on Capital (sunk in Aircraft and Spares) — $0{\cdot}03\frac{C}{U}$

HOURLY COSTS

4. *Maintenance and Overhaul*
   4.1. Airframe and Equipment and Engine Maintenance — $3{\cdot}0\times{}^{-4}W_e$
   (W_e = equipped weight less engines and rotor systems and transmission)
   4.2. Rotor Systems and transmissions — $0{\cdot}015np_e^{2/3}$
   (n = number of rotors per aircraft)
   (p_e = cruising power absorbed by each rotor (hp))
   4.3. *Engine Overhauls* — $12\frac{N}{L}P_e^{2/3}$
   (N = number of engines per aircraft)
   (P_e = cruising power per engine (hp))
   (L = overhaul life (hrs.))
5. *Flying Crew Costs* — 6·0
6. *Fuel and Oil*
   Makers' consumptions + 3 per cent. for a stage duration in 4 kt. headwind plus 0·1 hr. to allow for warm-up and terminal lost time.
   6·1 Fuel cost at £0·23 per gal. (petrol)
   £0·10 per gal. (kerosene)
   6·2 Oil cost at £0·3 per gal.

LANDING COSTS

7. *Landing Fees*
   0- 35 n. mls. $0{\cdot}25\times10^{-4}Wg$ £/Landing
   35- 70 n. mls. $0{\cdot}50\times10^{-4}Wg$ £/Landing
   70-100 n. mls. $0{\cdot}625\times10^{-4}Wg$ £/Landing
   100 n. mls. $1{\cdot}25\times10^{-4}Wg$ £/Landing
   (Wg = gross weight in lb.)

APPENDIX IV

	Pure Helicopter		*Compound Helicopter*		*Convertible Helicopter*	*Turboprop*	*Subsonic Jet*	*Supersonic Jet*
	Low Solidity	*High Solidity*	*Low Solidity*	*High Solidity*				
Equipped airframe	12,916	12,899	14,912	14,601	15,178	19,290	18,170	20,600
Blades	1,560	2,520	1,260	1,735	1,210	640	0	0
Hubs and transmission	5,025	5,200	4,250	4,490	2,282	740	0	0
Power plant	6,523	6,523	6,523	6,523	16,533	7,613	7,870	12,400
Crew	554	554	554	554	554	554	554	380
Disposable load	23,422	22,304	22,501	22,097	14,243	21,163	23,406	16,620
All-up weight	50,000	50,000	50,000	50,000	50,000	50,000	50,000	50,000
				WEIGHT BREAKDOWN lb.				
Equipped airframe	10·0	10·0	10·0	10·0	10·0	10·0	10·0	16·3*
Blades	15·0	15·0	15·0	15·0	15·0	15·0	—	—
Hubs and Transmission	5·0	5·0	5·0	5·0	5·0	5·0	—	—
Power plant	10·0	10·0	10·0	10·0	10·0	10·0	10·0	10·0
Complete aircraft	9·33	9·50	9·45	9·50	9·87	10·00	10·0	14·37
				FIRST COST £/lb.				
Equipped airframe	129,160	128,990	149,120	146,010	151,780	192,900	181,700	336,000
Blades	23,400	37,800	18,900	26,000	18,150	9,600	0	0
Hubs and Transmission	25,125	26,000	21,250	22,450	11,410	3,700	0	0
Power plant	65,230	65,230	65,230	65,230	165,330	76,130	78,700	124,000
Total cost	242,915	258,020	254,500	259,690	346,670	282,330	260,400	460,000
				FIRST COST £				

Assumed Production—100 off.
Prototype and development costs *not* included.

* Machined Skin.

The Analysis and Synthesis of Vibrating Systems

by

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Summary: Complicated oscillatory systems may be broken down into component "sub-systems" for the purpose of vibration analysis. These will generally submit more readily to analytical treatment. After an introduction to the concept of receptance, the principles underlying this form of analysis are reviewed.

The dynamical properties of simple systems (in the form of their receptances) may be tabulated. By this means the properties of a complicated system may be found by first analysing it into convenient sub-systems and then extracting the properties of the latter from a suitable table. A catalogue of this sort is given for the particular case of conservative torsional systems with finite freedom.

The properties of the composite system which may be readily found in this way are (*i*) its receptances and (*ii*) its frequency equation. Tables are given of expressions for these in terms of the receptances of the component sub-systems. All of the tables may easily be extended. The tabulated receptances may also be used for determining relative displacements during free vibration in any principal mode.

A method of presenting information on the vibration characteristics of machinery, which is effectively due to Carter[1], is illustrated by means of an example. More general adoption by manufacturers of this method (which requires no more computational effort than must normally be made) would lead to enormous savings of labour in calculating natural frequencies of composite systems.

1. Introduction

The theory of small oscillations of a linear system about a configuration of stable equilibrium is well established and has been in existence for many years (*e.g. see* Rayleigh[2]). Even when applied to relatively simple engineering systems, however, the theory is often cumbersome and a large body of literature is devoted to the important task of making vibration analysis more manageable.

An important step was made in this process of simplification when Carter[1] introduced the concept of a "dynamic flexibility." Additions have been made to his theory and an account of it, with the relevant references and many extensions, is given by Duncan[3] (*see also* Johnson[4]). The theory is now known as that of "receptance;" a previous title—of "mechanical admittance"—has been discarded owing to the possibility of its confusion with electrical terminology.

The receptances of a system, as will be seen, have a physical significance; they are measures of the readiness of the system to respond to harmonic exciting forces. On account of this there is much to be said, not merely for using them to simplify the book-keeping of standard vibration theory, but actually to develop that theory from first principles in terms of them. For by this means it is easier to keep in touch with physical reality.

A main advantage of the receptance technique is that it permits complicated systems to be broken down into simpler parts for the purposes of vibration analysis. These can be examined separately, either by test or by calculation or both. This type of analysis is the subject of the present paper and, for the sake of explanation, torsional oscillations of conservative systems with finite freedom (rigid discs and light shafts) are dealt with mainly.

The objects of this article are twofold. The first is to draw the attention of engineers to the receptance method by presenting a simple exposition of the fundamentals involved. This has, in fact, been done before by the authors mentioned; but their work does not appear to have received proper recognition. An alternative, and more recently proposed, technique is due to Firestone[5] and is known as the "mobility method"; this has already found a place in certain elementary American textbooks (for instance, *see* that of Hansen and Chenea[6]). The author believes that the earlier method is superior to that of mobility although, to be sure, either process is of great value in the analysis of complicated systems.

A method of presenting the vibration characteristics of engines, compressors, pumps, and so on will be illustrated. This is essentially due to Carter[1]* and, although it is simple to use, very considerable savings of time and effort can be made by its adoption.

The author's second object is to present certain tables of results. It has been pointed out that the use of receptances permits complex oscillatory systems to be analysed piece by piece. It is possible then, to catalogue results for the simple "sub-systems" so found.

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**See also* Johnson[4].

Notation

A composite system
$B, C, \ldots$ sub-systems which, together, constitute A
F amplitude of force or torque
F' inertia force or torque
f flexibility coefficient (*see* Appendix I)
I polar moment of inertia
k extensional or torsional stiffness
M mass
m mass
n number of degrees of freedom
P static force (*see* Appendix I)
t time
X amplitude of displacement in translation or rotation
x displacement (of translation or rotation)
α receptance of A
$\beta, \gamma \ldots$ receptances of $B, C, \ldots$
Δ denominator of expression for receptance. (*see* Table I)
δ static displacement (*see* Appendix I)
ω excitation frequency

2. Receptances

Let a torque $F \sin \omega t$ act upon a rigid disc which is free to rotate as in Fig. 1. The ang'e of rotation x is given by

$$I\ddot{x} = F \sin \omega t. \quad . \quad . \quad . \quad (1)$$

If attention is confined to variation of x with the excitation frequency ω, it is found by substituting

$$x = \text{constant} \times \sin \omega t$$

into (1) that

$$x = -\frac{1}{I\omega^2} F \sin \omega t. \quad . \quad . \quad (2)$$

This result may be written in the form

$$x = \alpha F \sin \omega t \quad . \quad . \quad . \quad (3)$$

where $\alpha (= -1/I\omega^2)$ is called "direct receptance at x."

In the same way, if a clamped massless shaft of torsional stiffness k has an axial torque $F \sin \omega t$ applied at its free end, as in Fig. 2, the response at the free end is given by

$$kx = F \sin \omega t. \quad . \quad . \quad . \quad (4)$$

That is, the receptance α (of equation (3)) at the free end is equal to $1/k$.

Consider now the system of Fig. 3 which has two degrees of freedom. If a harmonic torque $F_1 \sin \omega t$ is applied at x_1, the equations of motion are

$$\left.\begin{aligned} k(x_1 - x_2) &= F_1 \sin \omega t \\ I\ddot{x}_2 &= k(x_1 - x_2) \end{aligned}\right\}. \quad . \quad . \quad . \quad (5)$$

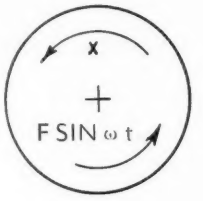


FIGURE 1. Harmonic excitation of a freely rotating disc.

In seeking solutions which have the forcing frequency, it is found by substitution that

$$\left.\begin{aligned} x_1 &= \left[-\frac{(k - I\omega^2)}{kI\omega^2} \right] F_1 \sin \omega t \\ x_2 &= \left[-\frac{1}{I\omega^2} \right] F_1 \sin \omega t. \end{aligned}\right\} \quad . \quad (6)$$

There are now two receptances; the first,

$$\alpha_{11} = -\frac{(k - I\omega^2)}{kI\omega^2} \quad . \quad . \quad . \quad (7)$$

is called the "direct receptance" at x_1, where the first subscript of α indicates the co-ordinate at which the displacement is measured while the second denotes the co-ordinate at which the harmonic forcing occurs. The second receptance is

$$\alpha_{21} = -\frac{1}{I\omega^2} \quad . \quad . \quad . \quad (8)$$

and this is called the "cross receptance" between x_1 and x_2.

If, instead of a torque $F_1 \sin \omega t$ as in Fig. 3, a torque $F_2 \sin \omega t$ acts at x_2, the equations are

$$\left.\begin{aligned} x_1 &= x_2 \\ I\ddot{x}_2 &= F_2 \sin \omega t \end{aligned}\right\}. \quad . \quad . \quad . \quad (9)$$

instead of (5). It follows, by the previous method that, now,

$$\left.\begin{aligned} \alpha_{22} &= -\frac{1}{I\omega^2} \\ \alpha_{12} &= -\frac{1}{I\omega^2}. \end{aligned}\right\} \quad . \quad . \quad (10)$$

It will be seen that $\alpha_{21} = \alpha_{12}$ in this example; it is a general property of vibrating systems that

$$\alpha_{rs} = \alpha_{sr} \quad . \quad . \quad . \quad (11)$$

as is shown in Appendix I. The fact that $\alpha_{12} = \alpha_{22}$ in this example has no such general significance, being merely an obvious property of the system under discussion.

In the same manner, the receptances of more complicated systems may be found by the process of substitution.

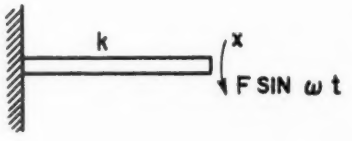


FIGURE 2. Harmonic torsional excitation of a massless uniform elastic shaft.

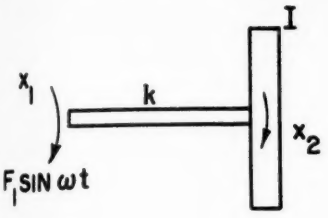


FIGURE 3. Harmonic torsional excitation of a system with two degrees of freedom.

In steady forced harmonic motion of a system, that part of the response which has the impressed frequency is almost unaffected by the existence of light damping except near resonance. On the other hand, vibration with the natural frequencies dies out. These conclusions are arrived at by examining the results of including damping forces in the equations of motion. It follows then that the part of the motion with the forcing frequency is the important one in steady oscillation and the receptance α_{rs} is a measure of the response at x_r due to a torque $F_s \sin \omega t$ applied at the co-ordinate x_s.

Table I gives a list of the receptances of some simple systems. These results have been found, in effect, by substitution into the relevant equations of motion in the manner already discussed. By putting in numerical values of the stiffnesses k and inertias I, information may be had on the responses of these systems to excitation at one of the co-ordinates.

Consider the motion of system number 4 in Table I due to an excitation $F_1 \sin \omega t$ at x_1. It is evident, from the form of α_{11}, that the closer the forcing frequency ω is to the value $\sqrt{(k/I)}$, the smaller need the amplitude F_1 be to maintain a finite motion. The resonance condition, where F_1 must be zero if the response is not to be indefinitely great, is given by an infinite value of the receptance—that is, when $\omega^2 = \omega_1^2 = k/I$.

Now all the receptances of any given system have the same denominator; this is denoted in the table by the symbol Δ. Therefore the condition

$$\Delta = 0 \quad . \quad . \quad . \quad (12)$$

implies that all the receptances are infinite and this is the condition of resonance. Equation (12) is a relation governing ω^2 in terms of the parameters of the system concerned. It may be shown to have as many roots as there are degrees of freedom in the system, all of which are real and positive (although one will be $\omega^2 = 0$ if the system is not anchored and is capable of motion as a rigid body). Equation (12) is the "frequency equation" whose roots are the squares of the natural frequencies. Thus, by inserting numerical values into the various expressions Δ and equating them to zero, simple algebraic equations are arrived at which can be solved in an elementary fashion by a computer.

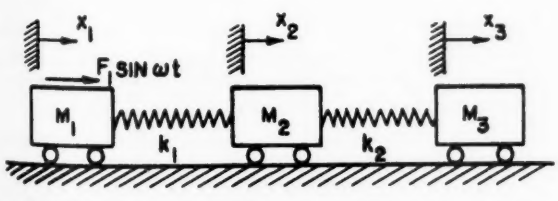


FIGURE 4. System of spring-connected masses executing forced oscillation.

3. Comments on the Theory of Receptances

The results given in Table I refer specifically to torsional oscillation of systems of rigid discs and light elastic shafts. These systems have been selected because of their technical importance. However the receptance functions listed are also immediately applicable to the axial vibrations of spring-connected masses. For instance, the cross receptance α_{13} of the system of Fig. 4 is obtained from system 11 of the table; it is

$$\alpha_{13} = k_1 k_2 / \Delta \quad . \quad . \quad . \quad (13)$$

where

$$\Delta = -\{M_1 M_2 M_3 \omega^6 - [k_1 (M_1 M_3 + M_2 M_3) + k_2 (M_1 M_2 + M_1 M_3)]\omega_4 + k_1 k_2 (M_1 + M_2 + M_3)\omega^2\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This is found simply by replacing I_1, I_2 and I_3 by M_1, M_2 and M_3 and using k_1 and k_2 to represent extensional, rather than torsional, stiffness.

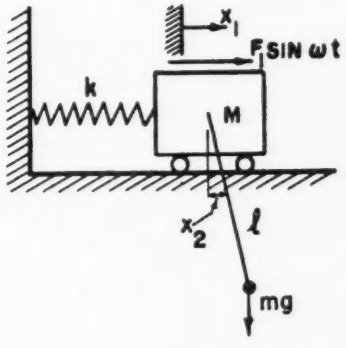


FIGURE 5. System with displacements of translation and rotation.

There is no need to differentiate in the theory of receptance between displacements x of translation and rotation or between forces and couples ($F \sin \omega t$). For instance, the equations of small forced oscillations of the system of Fig. 5 are found by elementary means to be

$$\left.\begin{aligned} M\ddot{x}_1 + kx_1 - mgx_2 &= F_1 \sin \omega t \\ \ddot{x}_1 + l\ddot{x}_2 + gx_2 &= 0. \end{aligned}\right\} \quad . \quad (15)$$

On substituting

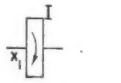
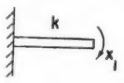
$$\left.\begin{aligned} x_1 &= X_1 \sin \omega t \\ x_2 &= X_2 \sin \omega t \end{aligned}\right\} \quad . \quad . \quad . \quad (16)$$

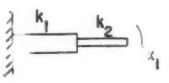
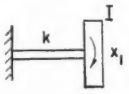
where X_1 and X_2 are unknown constants, it is found that

$$\left.\begin{aligned} X_1 &= \left(\frac{g - l\omega^2}{(g - l\omega^2)(k - M\omega^2) - mg\omega^2}\right) F_1 = \alpha_{11} F_1 \\ X_2 &= \left(\frac{\omega^2}{(g - l\omega^2)(k - M\omega^2) - mg\omega^2}\right) F_1 = \alpha_{21} F_1 \end{aligned}\right\} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

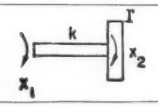
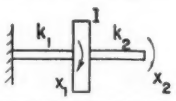
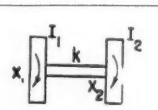
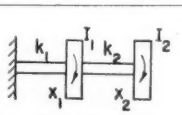
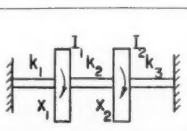
TABLE I

(*a*) RECEPTANCE FUNCTIONS OF TORSIONAL SYSTEMS HAVING ONE DEGREE OF FREEDOM

	1	2
SYSTEM		
α_{11}	$-\frac{1}{I\omega^2}$	$\frac{1}{k}$

	3	4
SYSTEM		
α_{11}	$\frac{k_1+k_2}{k_1 k_2}$	$\frac{1}{k-I\omega^2}$

(*b*) SYSTEMS HAVING TWO DEGREES OF FREEDOM

	SYSTEM	α_{11}	α_{22}	$\alpha_{12}=\alpha_{21}$
5		$\frac{(k-I\omega^2)}{\Delta}$	$\frac{k}{\Delta}$	$\frac{k}{\Delta}$
		$\Delta=-kI\omega^2$		
6		$\frac{k_2}{\Delta}$	$\frac{k_1+k_2-I\omega^2}{\Delta}$	$\frac{k_2}{\Delta}$
		$\Delta=k_2(k_1-I\omega^2)$		
7		$\frac{k-I_2\omega^2}{\Delta}$	$\frac{k-I_1\omega^2}{\Delta}$	$\frac{k}{\Delta}$
		$\Delta=I_1I_2\omega^4-k(I_1+I_2)\omega^2$		
8		$\frac{k_2-I_2\omega^2}{\Delta}$	$\frac{k_1+k_2-I_1\omega^2}{\Delta}$	$\frac{k_2}{\Delta}$
		$\Delta=I_1I_2\omega^4-[k_1I_2+k_2(I_1+I_2)]\omega^2+k_1k_2$		
9		$\frac{k_2+k_3-I_2\omega^2}{\Delta}$	$\frac{k_1+k_2-I_1\omega^2}{\Delta}$	$\frac{k_2}{\Delta}$
		$\Delta=I_1I_2\omega^4-[k_1I_2+k_2(I_1+I_2)+k_3I_1]\omega^2+$ $+(k_1k_2+k_2k_3+k_3k_1)$		

Thus a cross receptance α_{21} can be found, of the usual form, which relates *rotational* displacements x_2 to the *force* $F_1 \sin \omega t$. In order to develop the general theory of linear vibration in terms of receptances, it is necessary to interpret α in equations of the form (3) as relating *generalised* displacements to *generalised* harmonic forces. For further information on this subject, reference should be made to Duncan[3] and Johnson[4]; in the interest of simplicity, attention will be confined to torsional systems here.

By introducing the notion of a *complex* receptance function, account may be taken of linear damping[3]. This, again, will not be dealt with here.

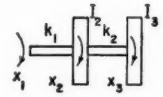
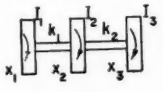
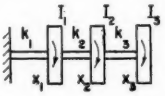
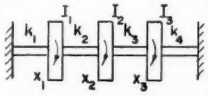
4. Analysis of Oscillating Systems into Component Sub-systems

A complicated vibratory system can be regarded as if it were built up from simpler "sub-systems" which are coupled together at co-ordinates. For instance system *A* of Fig. 6 can be regarded as a combination of systems *B* and *C* coupled together at x_1.

The process by which the oscillatory characteristics of a complex system can be investigated through those of its component parts will now be examined. In doing this, a complex system *A* will be said to have receptances α (with appropriate subscripts). The simpler

TABLE I

(*c*) SYSTEMS HAVING THREE DEGREES OF FREEDOM

10		α_{11}	$\{I_2I_3\omega^4-[k_1I_3+k_2(I_2+I_3)]\omega^2+k_1k_2\}/\Delta$
		α_{22}	$\{k_1(k_2-I_3\omega^2)\}/\Delta$
		α_{33}	$\{k_1(k_2-I_2\omega^2)\}/\Delta$
		α_{12} α_{21}	$\{k_1(k_2-I_3\omega^2)\}/\Delta$
		α_{13} α_{31}	$\{k_1k_2\}/\Delta$
		α_{23} α_{32}	$\{k_1k_2\}/\Delta$
		Δ	$k_1I_2I_3\omega^2-k_1k_2(I_2+I_3)\omega^2$
11		α_{11}	$\{I_2I_3\omega^4-[k_1I_3+k_2(I_2+I_3)]\omega^2+k_1k_2\}/\Delta$
		α_{22}	$\{I_1I_3\omega^4-(k_1I_3+k_2I_1)\omega^2+k_1k_2\}/\Delta$
		α_{33}	$\{I_1I_2\omega^4-[k_1(I_1+I_2)+k_2I_1]\omega^2+k_1k_2\}/\Delta$
		α_{12} α_{21}	$\{k_1(k_2-I_3\omega^2)\}/\Delta$
		α_{13} α_{31}	$\{k_1k_2\}/\Delta$
		α_{23} α_{32}	$\{k_2(k_1-I_1\omega^2)\}/\Delta$
		Δ	$-\{I_1I_2I_3\omega^4-[k_1(I_1I_3+I_2I_3)+k_2(I_1I_2+I_1I_3)]\omega^2+k_1k_2(I_1+I_2+I_3)\omega^2\}$
12		α_{11}	$\{I_2I_3\omega^4-[k_2I_3+k_3(I_2+I_3)]\omega^2+k_2k_3\}/\Delta$
		α_{22}	$\{I_1I_3\omega^4-[k_1I_3+k_2I_3+k_3I_1]\omega^2+k_1k_3+k_2k_3\}/\Delta$
		α_{33}	$\{I_1I_2\omega^4-[k_1I_2+k_2(I_1+I_2)+k_3I_1]\omega^2+k_1k_2+k_2k_3+k_3k_1\}/\Delta$
		α_{12} α_{21}	$\{k_2(k_3-I_3\omega^2)\}/\Delta$
		α_{13} α_{31}	$\{k_2k_3\}/\Delta$
		α_{23} α_{32}	$\{k_3(k_1+k_2-I_1\omega^2)\}/\Delta$
		Δ	$-\{I_1I_2I_3\omega^6-[k_1I_2I_3+k_2(I_1I_3+I_2I_3)+k_3(I_1I_2+I_1I_3)]\omega^4+$ $+[I_1k_2k_3+I_2(k_1k_3+k_2k_3)+I_3(k_1k_2+k_2k_3+k_1k_3)]\omega^2-k_1k_2k_3\}$
13		α_{11}	$\{I_2I_3\omega^4-[k_2I_3+k_3(I_2+I_3)+k_4I_2]\omega^2+k_2k_3+k_3k_4+k_4k_2\}/\Delta$
		α_{22}	$\{I_1I_3\omega^4-[k_1I_3+k_2I_3+k_3I_1+k_4I_1]\omega^2+(k_1+k_2)(k_3+k_4)\}/\Delta$
		α_{33}	$\{I_1I_2\omega^4-[k_1I_2+k_2(I_1+I_2)+k_3I_1]\omega^2+k_1k_2+k_2k_3+k_3k_1\}/\Delta$
		α_{12} α_{21}	$\{k_2(k_3+k_4-I_3\omega^2)\}/\Delta$
		α_{13} α_{31}	$\{k_2k_3\}/\Delta$
		α_{23} α_{32}	$\{k_3(k_1+k_2-I_1\omega^2)\}/\Delta$
		Δ	$-\{I_1I_2I_3\omega^6-[k_1I_2I_3+k_2(I_1I_3+I_2I_3)+k_3(I_1I_2+I_1I_3)+k_4I_1I_2]\omega^4+$ $+[I_1(k_2k_3+k_3k_4+k_4k_2)+I_2(k_1k_3+k_3k_2+k_2k_4+k_4k_1)+$ $+I_3(k_1k_2+k_2k_3+k_3k_1)]\omega^2-[k_1k_2(k_3+k_4)+k_3k_4(k_1+k_2)]\}$

systems *B*, *C* . . . (which together make up *A*) will be said to have receptances β, γ . . . Thus in the example of Fig. 6, Table I shows that

$$\left.\begin{aligned}\beta_{11}&=-\frac{1}{I_1\omega^2}\\ \gamma_{11}&=\frac{k-I_2\omega^2}{-kI_2\omega^2}\\ \gamma_{22}&=-\frac{1}{I_2\omega^2}\\ \gamma_{12}&=-\frac{1}{I_2\omega^2}\end{aligned}\right\} \quad . \quad . \quad . \quad (18)$$

The receptances α_{11}, α_{22}, α_{12} are those of system number 7 in Table I.

The theory that will now be treated applies to any type of linear vibrating system and is not confined to the torsional systems of Table I, provided that *generalised* forces and *generalised* displacements are used. Accordingly, it is convenient to represent the component parts *B*, *C* . . . in the "block" form shown in Table II. The lines connecting the various systems represent the co-ordinates x_1, x_2, and so on, at which they are joined. Thus the block diagram appropriate to Fig. 6 is that shown in Fig. 6(*c*).

Oscillations of any composite system *A*, whether

free or forced, place simple requirements upon the motions of its sub-systems *B, C* . . . ; these requirements may be termed those of "equilibrium" and "compatibility." Before dealing with these, however, it is necessary to examine the principle of "superposition."

(*i*) *Superposition.* By hypothesis only those systems are contemplated whose small forced oscillations are governed by linear equations with constant coefficients. It follows that the principle of superposition applies to them. That is, if harmonic excitations are applied at two or more points of a system, the response is everywhere the sum of those due to the excitations as if they were applied separately. This is a property of the differential equations of motion.

Let harmonic torques $F_1 \sin \omega t, F_2 \sin \omega t, \ldots F_n \sin \omega t$ be applied at the co-ordinates $x_1, x_2, \ldots x_n$ of a torsional system with n degrees of freedom. If only those vibrations are considered which have the excitation frequency ω, the responses at the various co-ordinates are given by

$$\left.\begin{aligned} x_1 &= \alpha_{11}F_1 \sin \omega t + \alpha_{12}F_2 \sin \omega t + \ldots + \alpha_{1n}F_n \sin \omega t \\ x_2 &= \alpha_{21}F_1 \sin \omega t + \alpha_{22}F_2 \sin \omega t + \ldots + \alpha_{2n}F_n \sin \omega t \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n &= \alpha_{n1}F_1 \sin \omega t + \alpha_{n2}F_2 \sin \omega t + \ldots + \alpha_{nn}F_n \sin \omega t \end{aligned}\right\} \quad (19)$$

If $F_1 \neq 0$ and $F_2 = F_3 = \ldots = F_n = 0$, then this equation gives the variations of $x_1, x_2 \ldots x_n$ according to the definitions of $\alpha_{11}, \alpha_{21} \ldots \alpha_{n1}$. Similarly, if $F_2 \neq 0$ and $F_1 = F_3 = F_4 = \ldots = F_n = 0$ then equation (19) gives the response to the torque $F_2 \sin \omega t$ applied at x_2; and so on. That these separate solutions can be added in this way is, of course, verifiable by direct substitution of (19) into the equation of motion for any particular system. The quantities $x_1, x_2 \ldots x_n$ are all proportional to $\sin \omega t$; this function may be omitted therefore by giving symbols $X_1, X_2 \ldots X_n$ to the amplitudes.

Results obtained previously for the system of Fig. 3 may be used to illustrate the meaning of (19). If the torques $F_1 \sin \omega t$ and $F_2 \sin \omega t$ are applied separately it has been shown that the motions are in accordance with the receptances

$$\alpha_{11} = -\frac{(k - I\omega^2)}{kI\omega^2} \quad ; \quad \alpha_{21} = -\frac{1}{I\omega^2} \quad . \quad (20)$$

and

$$\alpha_{12} = -\frac{1}{I\omega^2} \quad ; \quad \alpha_{22} = -\frac{1}{I\omega^2} \quad . \quad (21)$$

respectively. If, now, the torques are applied simultaneously, the equations are

$$\left.\begin{aligned} I\ddot{x}_1 &= k(x_1 - x_2) = F_1 \sin \omega t \\ I\ddot{x}_2 &= k(x_1 - x_2) + F_2 \sin \omega t. \end{aligned}\right\} \quad . \quad . \quad (22)$$

To find the response with the impressed frequency, the trial solutions

$$\left.\begin{aligned} x_1 &= X_1 \sin \omega t \\ x_2 &= X_2 \sin \omega t \end{aligned}\right\} \quad . \quad . \quad . \quad (23)$$

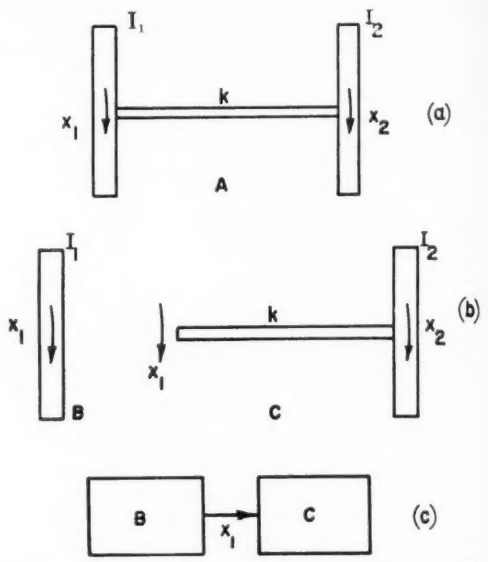


FIGURE 6. (*a*) A composite system *A*. (*b*) A possible set of simpler sub-systems *B* and *C* which, together, constitute *A*. (*c*) The block diagram of this system.

may be used; these yield

$$\left.\begin{aligned} X_1 &= -\frac{(k - I\omega^2)}{kI\omega^2}F_1 - \frac{1}{I\omega^2}F_2 = \alpha_{11}F_1 + \alpha_{12}F_2 \\ X_2 &= -\frac{1}{I\omega^2}F_1 - \frac{1}{I\omega^2}F_2 = \alpha_{21}F_1 + \alpha_{22}F_2. \end{aligned}\right\} \quad (24)$$

Thus the total motion is the sum of those due to the two torques separately.

It is a simple matter to show that the validity of equation (19) follows as a consequence of the postulated linearity of the equations of motion.

Equation (19) holds good, not only for *A*, but for the sub-systems *B, C*, . . . and, whenever a sub-system is considered, this will be evidenced by including the appropriate letter with the subscripts of the displacements and forces concerned. Thus, while (19) applies to *A*, the comparable equations for *B* are

$$\left.\begin{aligned} X_{b1} &= \beta_{11}F_{b1} + \beta_{12}F_{b2} + \ldots + \beta_{1m}F_{bm} \\ X_{b2} &= \beta_{21}F_{b1} + \beta_{22}F_{b2} + \ldots + \beta_{2m}F_{bm} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ X_{bm} &= \beta_{m1}F_{b1} + \beta_{m2}F_{b2} + \ldots + \beta_{mm}F_{bm} \end{aligned}\right\} \quad . \quad (25)$$

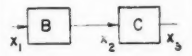
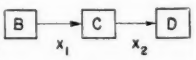
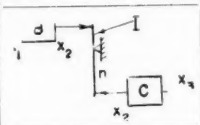
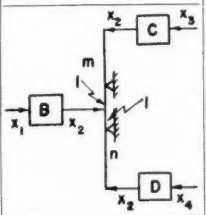
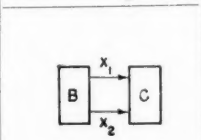
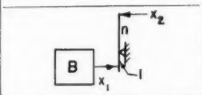
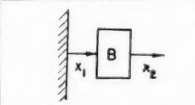
assuming that *B* has *m* degrees of freedom.

(*ii*) *Equilibrium.* Consider the sub-systems *B* and *C* of Fig. 6(*a*) which are connected at x_1. The torques exerted upon *B* and *C* at that co-ordinate must either be in equilibrium with an applied torque $F_1 \sin \omega t$ or, in the absence of excitation at x_1, between themselves. That is,

$$F_{b1} + F_{c1} = F_1 \quad . \quad . \quad . \quad (26)$$

TABLE II

SYNTHESIS OF RECEPTANCES

	SYSTEM	RECEPTANCE		
1		α_{11}	α_{22}	α_{33}
		$\beta_{11}-\dfrac{\beta_{12}^2}{\beta_{22}+\gamma_{22}}$	$\dfrac{\beta_{22}\gamma_{22}}{\beta_{22}+\gamma_{22}}$	$\gamma_{33}-\dfrac{\gamma_{23}^2}{\beta_{22}+\gamma_{22}}$
		$\alpha_{12}=\alpha_{21}$	$\alpha_{13}=\alpha_{31}$	$\alpha_{23}=\alpha_{32}$
		$\dfrac{\beta_{12}\gamma_{22}}{\beta_{22}+\gamma_{22}}$	$\dfrac{\beta_{12}\gamma_{23}}{\beta_{22}+\gamma_{22}}$	$\dfrac{\beta_{22}\gamma_{23}}{\beta_{22}+\gamma_{22}}$
2		α_{11}	$\dfrac{\beta_{11}[\gamma_{11}(\gamma_{22}+\delta_{22})-\gamma_{12}^2]}{(\beta_{11}+\gamma_{11})(\gamma_{22}+\delta_{22})-\gamma_{12}^2}$	
		α_{12} α_{21}	$\dfrac{\beta_{11}\gamma_{12}\delta_{22}}{(\beta_{11}+\gamma_{11})(\gamma_{22}+\delta_{22})-\gamma_{12}^2}$	
3		α_{11}	α_{33}	$\alpha_{13}=\alpha_{31}$
		$\beta_{11}-\dfrac{n^2\beta_{12}^2}{n^2\beta_{22}+\gamma_{22}}$	$\gamma_{33}-\dfrac{\gamma_{23}^2}{n^2\beta_{22}+\gamma_{22}}$	$\dfrac{n\beta_{12}\gamma_{23}}{n^2\beta_{22}+\gamma_{22}}$
4		α_{11}		α_{33}
		$\beta_{11}-\dfrac{\beta_{12}^2(n^2\gamma_{22}+m^2\delta_{22})}{\gamma_{22}\delta_{22}+\beta_{22}(n^2\gamma_{22}+m^2\delta_{22})}$		$\gamma_{33}-\dfrac{\gamma_{23}^2(n^2\beta_{22}+\delta_{22})}{\gamma_{22}\delta_{22}+\beta_{22}(n^2\gamma_{22}+m^2\delta_{22})}$
		$\alpha_{13}=\alpha_{31}$		$\alpha_{34}=\alpha_{43}$
		$\dfrac{mn\beta_{12}\gamma_{23}\delta_{22}}{\gamma_{22}\delta_{22}+\beta_{22}(n^2\gamma_{22}+m^2\delta_{22})}$		$\dfrac{mn\beta_{22}\gamma_{23}\delta_{24}}{\gamma_{22}\delta_{22}+\beta_{22}(n^2\gamma_{22}+m^2\delta_{22})}$
5		α_{11}	$\{\beta_{11}(\gamma_{11}\gamma_{22}-\gamma_{12}^2)+\gamma_{11}(\beta_{11}\beta_{22}-\beta_{12}^2)\}/\Delta$	
		α_{22}	$\{\beta_{22}(\gamma_{11}\gamma_{22}-\gamma_{12}^2)+\gamma_{22}(\beta_{11}\beta_{22}-\beta_{12}^2)\}/\Delta$	
		α_{12} α_{21}	$\{\beta_{12}(\gamma_{11}\gamma_{22}-\beta_{12}\gamma_{12})+\gamma_{12}(\beta_{11}\beta_{22}-\beta_{12}\gamma_{12})\}/\Delta$	
		Δ	$(\beta_{11}+\gamma_{11})(\beta_{22}+\gamma_{22})-(\beta_{12}+\gamma_{12})^2$	
6		$\alpha_{22}=n^2\beta_{22}$		
7		$\alpha_{22}=\beta_{22}-\dfrac{\beta_{12}^2}{\beta_{11}}$		

where F_1 may or may not be zero. This is a "condition of equilibrium."

System number 3 of Table II has a gear ratio between B and C. The equilibrium condition in this case is, in the absence of excitation at the junction x_2,

$$F_{b2}\times 1+F_{c2}\times n=0. \quad . \quad . \quad (27)$$

For the branched geared system (number 4 in the Table II) equilibrium demands that

$$F_{b2}+mF_{c2}+nF_{d2}=0 \quad . \quad . \quad (28)$$

if no excitation exists at the branch point.

In system 5 of Table II, A is composed of two parts B and C which are linked at two co-ordinates x_1 and x_2. Suppose, for the sake of argument, that a harmonic force acts at x_2 only; the conditions of equilibrium are then

$$\left.\begin{aligned} F_{b1}+F_{c1}&=0\\ F_{b2}+F_{c2}&=F_2. \end{aligned}\right\} \quad . \quad . \quad (29)$$

(*iii*) *Compatibility.* It is clear that, if B and C of Fig. 6 are to remain coupled,

$$X_{b1}=X_{c1}. \quad . \quad . \quad . \quad (30)$$

This is a simple "equation of compatibility;" it is concerned with the *continuity* of the composite system A.

The compatibility requirement of the geared system (3 of Table II) is

$$nX_{b2}=X_{c2} \quad . \quad . \quad . \quad (31)$$

while that of the branched system (number 4) is

$$\left.\begin{aligned} mX_{b_2} &= X_{c_2} \\ nX_{b_2} &= X_{d_2}. \end{aligned}\right\} \quad . \quad . \quad . \quad (32)$$

Finally, that of the doubly linked system (number 5) is

$$X_{b_1} = X_{c_1}; \qquad X_{b_2} = X_{c_2}. \quad . \quad . \quad (33)$$

By applying conditions of equilibrium and compatibility to equations of the form (25), it is possible to find overall receptances α of a composite system in terms of the receptances β, γ . . . of its component parts. A selection of these results appears in Table II.

To illustrate the procedure, consider system 2 in Table II. The equations of motion are

$$\left.\begin{aligned} X_{b_1} &= \beta_{11}F_{b_1} \\ X_{c_1} &= \gamma_{11}F_{c_1} + \gamma_{12}F_{c_2} \\ X_{c_2} &= \gamma_{21}F_{c_1} + \gamma_{22}F_{c_2} \\ X_{d_2} &= \delta_{22}F_{d_2}. \end{aligned}\right\} \quad . \quad . \quad (34)$$

If it be supposed that the only applied excitation is $F_1 \sin \omega t$ (acting at x_1), the equilibrium condition is

$$\left.\begin{aligned} F_{b_1} + F_{c_1} &= F_1 \\ F_{c_2} + F_{d_2} &= 0. \end{aligned}\right\} \quad . \quad . \quad . \quad (35)$$

Compatibility demands that

$$\left.\begin{aligned} X_{b_1} &= X_{c_1} \\ X_{c_2} &= X_{d_2}. \end{aligned}\right\} \quad . \quad . \quad . \quad (36)$$

With these requirements, equations (34) yield

$$\left.\begin{aligned} X_{b_1} &= \left(\frac{\beta_{11}[\gamma_{11}(\gamma_{22}+\delta_{22}) - \gamma_{12}^2]}{(\beta_{11}+\gamma_{11})(\gamma_{22}+\delta_{22}) - \gamma_{12}^2}\right) F_1 = \alpha_{11}F_1 \\ X_{d_2} &= \left(\frac{\beta_{11}\gamma_{12}\delta_{22}}{(\beta_{11}+\gamma_{11})(\gamma_{22}+\delta_{22}) - \gamma_{12}^2}\right) F_1 = \alpha_{12}F_2 \end{aligned}\right\} \quad (37)$$

in which the fact that $\gamma_{12} = \gamma_{21}$ and $\alpha_{12} = \alpha_{21}$ has been taken into account. These values of $\alpha_{11} = \alpha_{12}$ appear in Table II.

The object of expressing receptances α in terms of β, γ . . . is that B, C . . . are usually more easily analysed than A. For instance, it may be convenient to measure receptances of the sub-systems experimentally and to combine the results to find the receptances α. It may also be expedient to make a catalogue of receptance functions (as in Table I) and to use this as a dictionary of results.

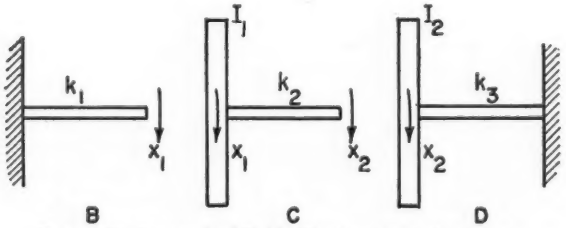


FIGURE 7. System 9 of Table I broken down into three sub-systems.

As an example, suppose that α_{12} of system 9 (Table I) is required. The system may be regarded as in Fig. 7. Using the results tabulated for systems 2, 5 and 4, it is seen that

$$\left.\begin{aligned} \beta_{11} &= \frac{1}{k_1} \\ \gamma_{11} &= -\frac{1}{I_1\omega^2} \\ \gamma_{22} &= \frac{k_2 - I_1\omega^2}{-k_2 I_1\omega^2} \\ \gamma_{12} &= -\frac{1}{I_1\omega^2} \\ \delta_{22} &= \frac{1}{k_3 - I_2\omega^2} \end{aligned}\right\} \quad . \quad (38)$$

These quantities may be substituted into the expression for α_{12} quoted in Table II (system 2). It is then found that α_{12} of system 9 (Table I) is arrived at after some reduction.

In general, this application of Table II is not worthwhile, as it is easier to find the analytical expressions for receptances directly by systematic use of the method of substitution into the equations of motion. But, if numerical values are available so that the various receptances of the sub-systems can be evaluated (by experiment or by the use of a catalogue like Table I), the saving of time and labour may well become considerable.

5. Frequency Equations

A method has been shown of finding the frequency equation (*see* equation (12)). It was to find any receptance of the system, whence the natural frequencies were given by the condition that this receptance should be infinite. A second, very useful, method of finding these frequencies is suggested by the technique of breaking a complex system A down into its component sub-systems B, C . . .

The condition that a receptance α (of A) shall be infinite is that the denominator of an expression like those in Table II shall vanish. Thus, the frequency equation of the geared system (number 3) is

$$n^2\beta_{22} + \gamma_{22} = 0. \quad . \quad . \quad . \quad (39)$$

Therefore, if β_{22} and γ_{22} are known functions of ω—found either by experiment or by calculation, or by the use of a catalogue such as Table I—then the natural frequencies of the geared system can be found. It may be convenient, for instance, to plot $n^2\beta_{22}$ and $-\gamma_{22}$ against ω and to find the values of ω at which the curves intersect.

A number of frequency equations of this sort are given in Table III. It will be seen that, in most cases, the relations follow from the results in Table II. For example, by putting $\gamma_{12} = \gamma_{22} = \gamma_{12} = 0$ in the quantity Δ of system 5 of Table II and equating Δ to zero, the frequency equation of system 2 (Table III) is found.

To illustrate the use of Table III, consider the branched system of Fig. 8 for which

$k_1=3 \times 10^6$ lb. in./rad., $I_1=4$ lb. in. sec.^{2} or 1,536 lb. in.^{2}
$k_2=1 \times 10^6$ lb. in./rad., $I_2=16$ lb. in. sec.^{2} or 6,144 lb. in.^{2}
$k_3=2 \times 10^6$ lb. in./rad., $I_3=12$ lb. in. sec.^{2} or 4,608 lb. in.^{2}
$k_4=1{\cdot}8 \times 10^6$ lb. in./rad., $I_4=10$ lb. in. sec.^{2} or 3,840 lb. in.^{2}

Suppose that the speed ratio between the drive shaft and axles is 4 to 1. Using the sub-systems *B*, *C* and *D* shown, equation (14) of Table III gives the frequency equation

$$\gamma_{11}\delta_{11}+\beta_{11}\left[\left(\frac{1}{4}\right)^2\gamma_{11}+\left(\frac{1}{4}\right)^2\delta_{11}\right]=0. \qquad (40)$$

From 10 of Table I,

$$\beta_{11}=\frac{16\times 4\omega^4-[1\times 10^6\times 4+3\times 10^6(16+4)]\omega^2+3\times 10^{12}}{1\times 10^6\times 16\times 4\omega^4-3\times 10^{12}(16+4)\omega^2}$$

while from 5 of Table I,

$$\gamma_{11}=\frac{2\times 10^6-12\omega^2}{-2\times 10^6\times 12\omega^2}\ ; \qquad \delta_{11}=\frac{1{\cdot}8\times 10^6-10\omega^2}{-1{\cdot}8\times 10^6\times 10\omega^2}.$$

That is, equation (40) gives an algebraic equation in ω^2 which can be solved by standard means. The natural frequencies are, in fact:—

384 rad./sec.; 418 rad./sec.; 971 rad./sec.

It has been shown that the frequency equation of a system is found as the condition under which the receptances become infinite. The first method involved finding the quantity Δ in the expressions for receptance and equating it to zero (*see* equation (12)); by this means all of the natural frequencies may be determined. According to the second method, the frequency equation is the condition under which the receptances of the compound system *A*, *expressed in terms of the* β*'s,* γ*'s, and so on,* become infinite.

Under one special set of circumstances, the frequency equation found by this last method will not predict natural frequencies that actually exist. This is the case where, during free oscillation, no forces are exerted between the sub-systems; for it is implicit in the results of Table II, that all the forces at the junctions are not permanently zero in free motion. That this is so can be seen by supposing that the forces at the junctions are always zero in free oscillation. This would mean that, in the equations (25) for each sub-system, the *X*'s will be finite while the *F*'s all vanish; this, in turn, implies that receptances of *B*, *C* . . . must be infinite. If this is so, there is no assurance that the equations of Table III represent conditions under which the receptances of *A* become infinite.

Suppose that *B*, *C* . . . all have a common natural frequency and that adjacent sub-systems are connected at only one co-ordinate. They can oscillate with this frequency and no forces will exist at the junctions. This natural frequency will not emerge from the frequency equation as found by the second method.

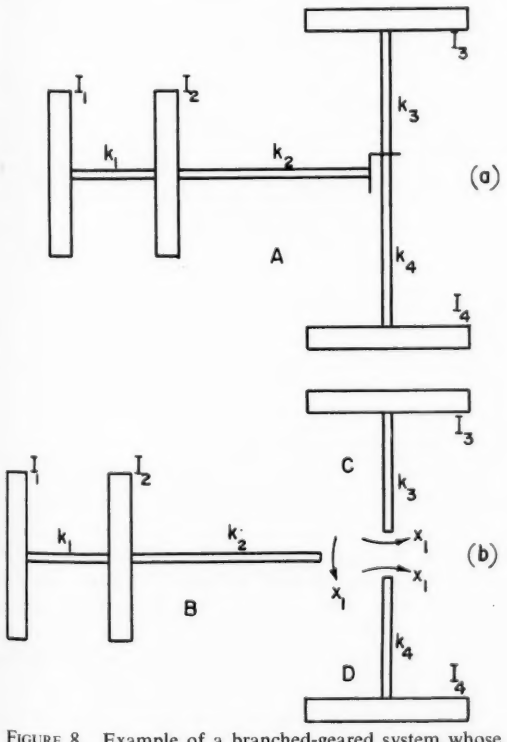


FIGURE 8. Example of a branched-geared system whose frequency equation is to be found. (*a*) The composite system *A*. (*b*) Sub-systems *B*, *C* and *D* which, together, constitute *A*.

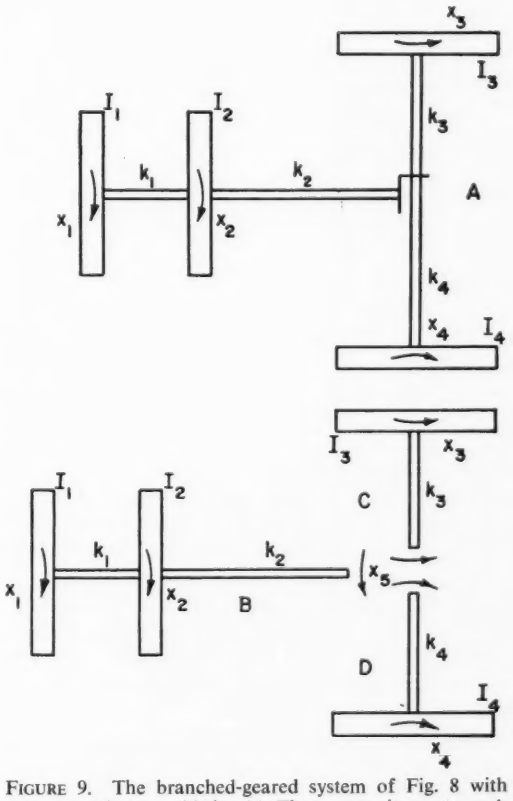
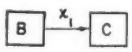
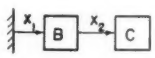
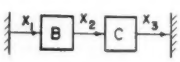
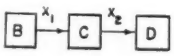
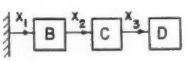
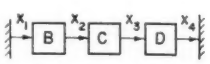
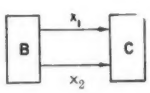


FIGURE 9. The branched-geared system of Fig. 8 with new co-ordinates added. (*a*) The composite system *A*. (*b*) Sub-systems *B*, *C* and *D* which, together, constitute *A*.

TABLE III

FREQUENCY EQUATIONS

	SYSTEM	FREQUENCY EQUATION
1		$\beta_{11}=0$
2		$\beta_{11}\beta_{22}-\beta_{12}^2=0$
3		$\beta_{11}+\gamma_{11}=0$
4		$\beta_{11}(\beta_{22}+\gamma_{22})-\beta_{12}^2=0$
5		$\beta_{11}\gamma_{33}(\beta_{22}+\gamma_{22})-(\beta_{11}\gamma_{23}^2+\beta_{12}^2\gamma_{33})=0$
6		$(\beta_{11}+\gamma_{11})(\gamma_{22}+\delta_{22})-\gamma_{12}^2=0$
7		$\beta_{11}[(\beta_{22}+\gamma_{22})(\gamma_{33}+\delta_{33})-\gamma_{23}^2]-\beta_{12}^2(\gamma_{33}+\delta_{33})=0$
8		$\beta_{11}\{(\beta_{22}+\gamma_{22})[\delta_{44}(\gamma_{33}+\delta_{33})-\delta_{34}^2]-\gamma_{23}^2\delta_{44}\}-$ $-\beta_{12}^2[\delta_{44}(\gamma_{33}+\delta_{33})-\delta_{34}^2]=0$
9		$(\beta_{11}+\gamma_{11})(\beta_{22}+\gamma_{22})-(\beta_{12}+\gamma_{12})^2=0$

Let two systems B and C be connected together at more than one co-ordinate. The conditions that no force shall appear at the links are (*a*) that they shall have a common natural frequency and (*b*) that, when oscillating at this frequency, the natural modes of B and C are such that compatibility is retained at all instants. The important case is obviously when identical systems are linked symmetrically. Here again, the equations of Table III would not give the appropriate natural frequency.

This lack of complete generality on the part of the second method of finding natural frequencies does not detract materially from its value. For the exceptional cases are easily identified and are easily calculated.

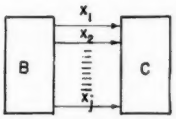
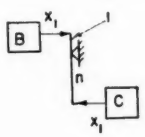
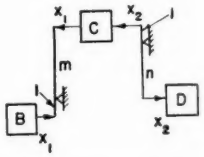
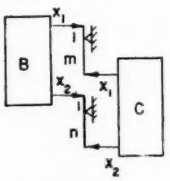
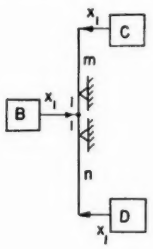
6. Free Oscillations

The principal modes in which a complicated system oscillates freely may usually be found most conveniently by arithmetical means as in the Holzer method. It is sometimes required, however, to find only the ratio of the displacements at two specified co-ordinates during free motion; in this case, the receptance technique may save much labour as it permits such a ratio to be found without requiring, first, a knowledge of the appropriate principal mode.

This may be illustrated with the system of Fig. 8 which is re-drawn in Fig. 9 with a fresh set of co-ordinates; of these, x_5 is not a true co-ordinate of the complete system A since the gears are supposed to be

TABLE III

FREQUENCY EQUATIONS

	SYSTEM	FREQUENCY EQUATION
10		$\begin{vmatrix} \beta_{11}+\gamma_{11} & \beta_{12}+\gamma_{12} \cdots\cdots \beta_{1j}+\gamma_{1j} \\ \beta_{21}+\gamma_{21} & \beta_{22}+\gamma_{22} \cdots\cdots \beta_{2j}+\gamma_{2j} \\ \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots \\ \beta_{j1}+\gamma_{j1} & \beta_{j2}+\gamma_{j2} \quad \beta_{jj}+\gamma_{jj} \end{vmatrix} = 0$
11		$n^2\beta_{11}+\gamma_{11}=0$
12		$(\gamma_{11}+m^2\beta_{11})(\delta_{22}+n^2\gamma_{22})-n^2\gamma_{12}^2=0$
13		$(m^2\beta_{11}+\gamma_{11})(n^2\beta_{22}+\gamma_{22})-(mn\beta_{12}+\gamma_{12})^2=0$
14		$\gamma_{11}\delta_{11}+\beta_{11}(n^2\gamma_{11}+m^2\delta_{11})=0$

light, but it is introduced for convenience at the junction of the sub-systems. If a harmonic torque $F_1 \sin \omega t$ is applied at x_1 the responses at x_3 and x_4 will be

$$\left.\begin{aligned} x_3 &= \alpha_{31}F_1 \sin \omega t \\ x_4 &= \alpha_{41}F_1 \sin \omega t \end{aligned}\right\} \quad . \quad . \quad . \quad (41)$$

and the appropriate cross receptances may be found from Table II (system 4). The ratio of these two displacements is given by the ratio of the numerators of α_{31} and α_{41}; that is,

$$\frac{x_3}{x_4} = \frac{\left(\frac{1}{4}\right)\beta_{15}\gamma_{53}\delta_{55}}{\left(\frac{1}{4}\right)\beta_{15}\delta_{54}\gamma_{55}} = \frac{\gamma_{53}\delta_{55}}{\delta_{54}\gamma_{55}} \quad . \quad . \quad (42)$$

The values of γ_{55}, γ_{53}, δ_{55}, δ_{54} are given by system number 5 of Table I so that, using the numerical values of the system,

$$\frac{x_3}{x_4} = \frac{\left(-\frac{1}{12\omega^2}\right)\left[-\frac{(1{\cdot}8\times10^6-10\omega^2)}{1{\cdot}8\times10^6\times10\omega^2}\right]}{\left(-\frac{1}{10\omega^2}\right)\left[-\frac{(2\times10^6-12\omega^2)}{2\times10^6\times12\omega^2}\right]} = \frac{3{\cdot}6\times10^6-20\omega^2}{3{\cdot}6\times10^6-21{\cdot}6\omega^2} \quad . \quad . \quad (43)$$

The ratio x_3/x_4 is seen to depend upon the excitation frequency ω. If this frequency is now taken closer and closer to one of the natural frequencies, both α_{31} and α_{41} of (41) become indefinitely great so that the forcing

amplitude F must approach zero if the motion is to remain finite. However, the numerators of these admittances remain finite and continue to determine the ratio x_3/x_4 as in equation (43). Thus, during *free* motion with the second natural frequency ($\omega=\omega_2=418$ rad./sec.),

$$\frac{x_3}{x_4}=\frac{3{\cdot}6\times10^6-20\times418^2}{3{\cdot}6\times10^6-21{\cdot}6\times418^2}=-\frac{0{\cdot}606}{1}. \qquad (44)$$

The same result is, of course, found if the vanishingly small harmonic torque is supposed to be applied at some co-ordinate other than x_1.

The method used to find (44) provides a means of finding the principal modes of the system once the natural frequencies have been determined. Thus, by using the second root of the frequency equation and employing the method already described, it is found that

$$\frac{x_1}{x_4}=-0{\cdot}057;\ \frac{x_2}{x_4}=-0{\cdot}044;\ \frac{x_3}{x_4}=-0{\cdot}606;\ \frac{x_4}{x_4}=1. \qquad (45)$$

That is, oscillation in the second principal mode is such that, at all times

$$x_1 : x_2 : x_3 : x_4=-0{\cdot}057 : -0{\cdot}044 : -0{\cdot}606 : 1. \qquad (46)$$

In general, however, this method is not the most convenient for the complete determination of principal modes, its chief use being in the derivation of relations like equation (44).

It is seen that, to find the relative displacements of points in a system during free oscillation by this method, it is necessary to know the receptances and the relevant natural frequency. In the example given, these receptances were known through those of the sub-systems B, C and D; but if the system appears in Table I, its receptances are known directly and the calculation is shortened accordingly. For instance, if it were required to find the ratio x_1/x_3 of the system number 11 in Table I during free motion in the highest mode, it is first necesary to find the greatest root ($\omega=\omega_3$) of the equation

$$I_1I_2I_3\omega^6-[k_1(I_1I_3+I_2I_3)+k_2(I_1I_2+I_1I_3)]\omega^4+ +k_1k_2(I_1+I_2+I_3)\omega^2=0. \qquad (47)$$

The required ratio is then found by substituting ω_3 for ω in the relation

$$\frac{x_1}{x_3}=\frac{\alpha_{12}}{\alpha_{32}}=\frac{k_1(k_2-I_3\omega^2)}{k_2(k_1-I_1\omega^2)} \qquad (48)$$

in which, for convenience, an evanescent torque $F_2\sin\omega t$ is supposed to act at x_2.

7. The Specification of Vibration Characteristics of Machines

In the development of the foregoing theory, various simple examples have been used as illustrations. In each case the receptance method is shown to be convenient although other methods—notably that of Holzer—could be used instead at the cost of extra labour. It is proposed now to discuss one more example, while this is of a type that frequently arises, and is therefore of practical significance, it represents a class of problems that cannot conveniently be handled by Holzer's technique—at least without the expenditure of great computational effort.

Fig. 10(*a*) shows the idealised dynamical form of a four-cylinder engine and flywheel. For the sake of explanation, it will be supposed that the inertias and stiffnesses indicated in the diagram have the following values

$I_1=1{,}536$ lb. in.^{2} $(=4$ lb. in. sec.$^2)$;
$I_2=I_3=I_4=I_5=384$ lb. in.$^2=(1$ lb. in. sec.$^2)$;
$k_1=k_2=k_3=k_4=2\times10^6$ in. lb./rad.

Holzer's method can, of course, be used to find the critical speeds of the engine; it can also be used to find the natural frequencies of the combined system when the engine B is coupled (through the co-ordinate x_1) to a load C as shown in Fig. 10(*b*), provided that details of C are known.

Now the maker of the engine will not generally be acquainted with details of all the uses to which his product will be put. Therefore, if it is to drive a compressor (say), he can only predict the critical speeds when he has the compressor data. Given this, he can carry out a vibration analysis (for instance, by Holzer's method) for the *whole* system. This may be a very tedious procedure, particularly if B and C are complicated or if a *range* of compressors were being examined for use with the one engine. The important point is that the engine manufacturer's *prior* knowledge of his own engine is usually of little use to him because each case involves starting a fresh calculation. The same observations apply, of course, to the component C.

A way is evidently required of presenting the engine data in such a form as to render unnecessary any further

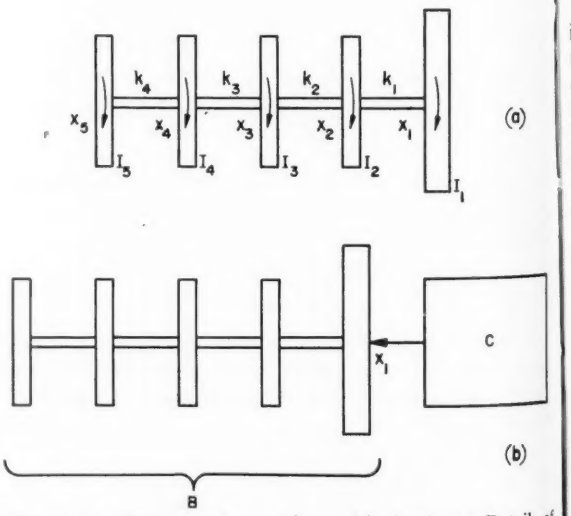


FIGURE 10. Example of an engine and its load. (*a*) Details of the engine. (*b*) The complete system A of engine B and load C.

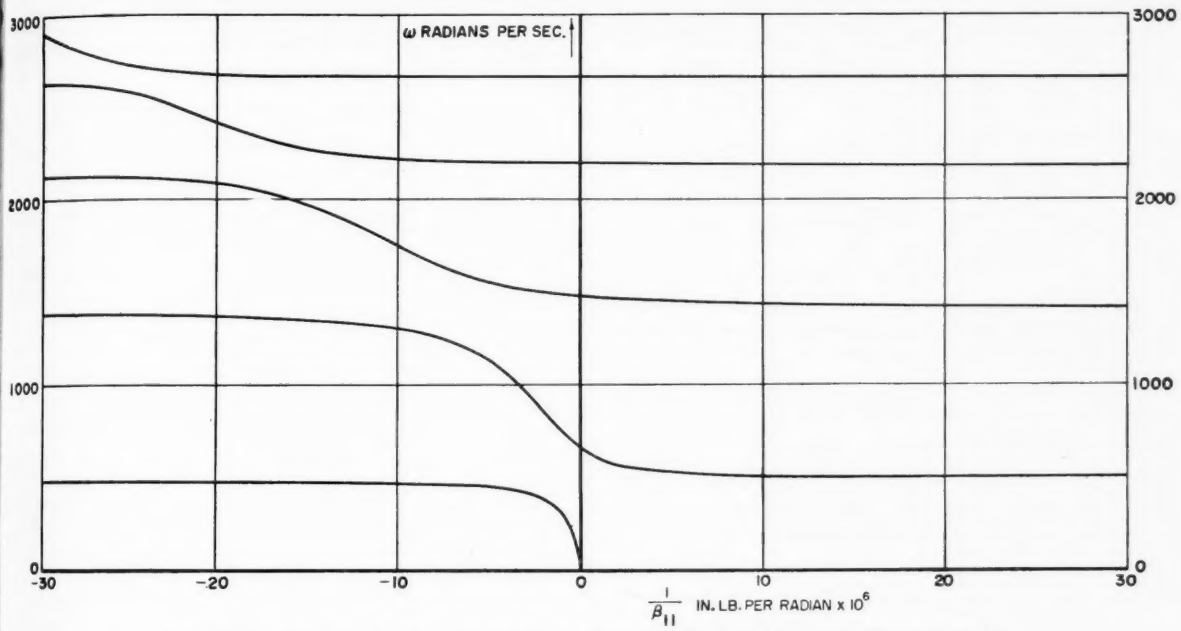


FIGURE 11. The curve showing the variation of the reciprocal of the receptance β_{11} with the excitation frequency ω for the system of Fig. 10(*a*).

calculation *as far as the engine is concerned*. It will now be shown that this can easily be done and that, if the same procedure were to be adopted by the maker of the system *C*, then no calculation is required to find the critical speeds of the combined system. Further, to present data in this form calls for no calculations beyond those which are normally made when using Holzer's method to find the natural frequencies of the unattached systems *B* and *C*. It is the author's opinion that, if the dynamical characteristics of machinery were presented by more manufacturers in the manner to be described (which is effectiveley due to Carter[1]) an enormous saving of labour would be achieved thereby.

The co-ordinate at which the engine *B* is attached to its load *C* is x_1 and attention will now be focused upon the receptance β_{11}. This quantity may be measured experimentally or found analytically, for any given value of ω, by one of the techniques already described. Alternatively it can be found by Holzer's method as shown in Appendix II and, in so far as this is normally done anyway in determining the natural frequencies of the uncoupled engine, it demands no extra labour. It is convenient to plot $1/\beta_{11}$ horizontally against ω vertically. This has been done for the system *B* (whose details have been given) and the curve is shown in Fig. 11.

It is worthwhile briefly to consider the form of the curve of $1/\beta_{11}$ as shown in Fig. 11 although a complete study of it is not necessary here*. The system *B* has five degrees of freedom so that it has fine natural frequencies at each one of which β_{11} is infinite, or $1/\beta_{11}=0$. These, then, correspond to the five points at which the curves meet the vertical axis; the lowest of these is at the origin since the system is capable of rotation as a rigid body and thus has a degenerate principal mode of zero frequency. Between the natural frequencies, there are frequencies at which $1/\beta_{11}$ becomes infinite (so that $\beta_{11}=0$). These frequencies are the so-called "anti-resonance" frequencies at which x_1 does not vary although the harmonic torque is applied at that co-ordinate; at these frequencies, *B* will oscillate freely if clamped at x_1.

The curve of Fig. 11 incorporates all the information that is needed concerning the engine when a vibration analysis is to be made of a complex system of which the engine forms part.

For the sake of explanation, the sub-system *C* of Fig. 10(*b*) has been selected as that shown in Fig. 12 in which

$I_6=I_7=1{,}920$ lb. in.^{2} $(=5$ lb. in. sec.$^2)$;

$k_6=1\times10^6$ in. lb./rad.; $\quad k_7=2\times10^6$ in. lb./rad.

Using these values, the variation of $1/\gamma_{11}$ with ω has been found and is shown plotted in Fig. 13. This curve is of the same general type as that of system *B* except that, at very high excitation frequencies

$$\frac{1}{\gamma_{11}}\longrightarrow k_6$$

or $$\gamma_{11}\longrightarrow 1\times10^{-6} \text{ rad./(in. lb.)}.$$

The reason for this is that, as ω becomes indefinitely large, the inertias I_6 and I_7 are stimulated less and less and the condition of Fig. 2 is approached. The curve of Fig. 13 embodies all that it is necessary to know

*A more general treatment is given by Johnson[4].

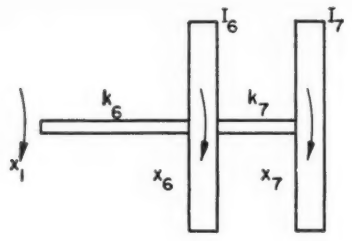


FIGURE 12. The sub-system C used in the example of Fig. 10(*b*).

about C in finding the natural frequencies of a compound system of which C is a sub-system; it might therefore be supplied by the maker of C.

It is now necessary to show how these curves of the reciprocal receptances can be used to determine the natural frequencies of the composite system A. According to entry number 1 of Table II,

$$\alpha_{11}=\beta_{11}\gamma_{11}/(\beta_{11}+\gamma_{11})$$

or

$$\frac{1}{\alpha_{11}}=\frac{1}{\beta_{11}}+\frac{1}{\gamma_{11}}. \quad . \quad . \quad . \quad (49)$$

At the natural frequencies of A, α_{11} is infinite so that

$$\frac{1}{\beta_{11}}=-\frac{1}{\gamma_{11}}. \quad . \quad . \quad . \quad (50)$$

That is, the natural frequencies of the complex system A are the values of ω at which the curve of $1/\beta_{11}$ intersects that of $-1/\gamma_{11}$. This is illustrated in Fig. 14 in which the curve of Fig. 13 is reversed and plotted on the same pair of axes as that of Fig. 11. The intersections of the sets of curves give the following approximate values of the natural frequencies

$\omega_1=0$; $\omega_2=386$ rad./sec.; $\omega_3=730$ rad./sec.;
$\omega_4=980$ rad./sec.; $\omega_5=1{,}480$ rad./sec.;
$\omega_6=2{,}190$ rad./sec.; $\omega_7=2{,}660$ rad./sec.

The higher values of the natural frequencies are quoted here merely for the sake of explanation; in practice they would be of no importance. At very high excitation frequencies, the basic assumption that an actual system can be analysed in terms of discs and shafts must be examined carefully.

The reader will observe that the frequency equation $\beta_{11}+\gamma_{11}=0$ is given in Table III so that β_{11} and γ_{11} may be plotted instead of the reciprocals. The equation

$$\beta_{11}=-\gamma_{11} \quad . \quad . \quad . \quad (51)$$

would then replace (50). The reason for not doing this in the foregoing example is simply that of convenience. It should be noted, however, that if the engine B is to drive some other component D (as well as C), the receptance at x_1 of B and C together is given more readily by curves of reciprocal receptances through equation (49).

8. Conclusions

One of the attractive features of introducing the idea of receptance into vibration theory is that it allows a complex system to be broken down into simpler components. These sub-systems will generally submit more readily to vibration analysis than will the main system from which they are taken.

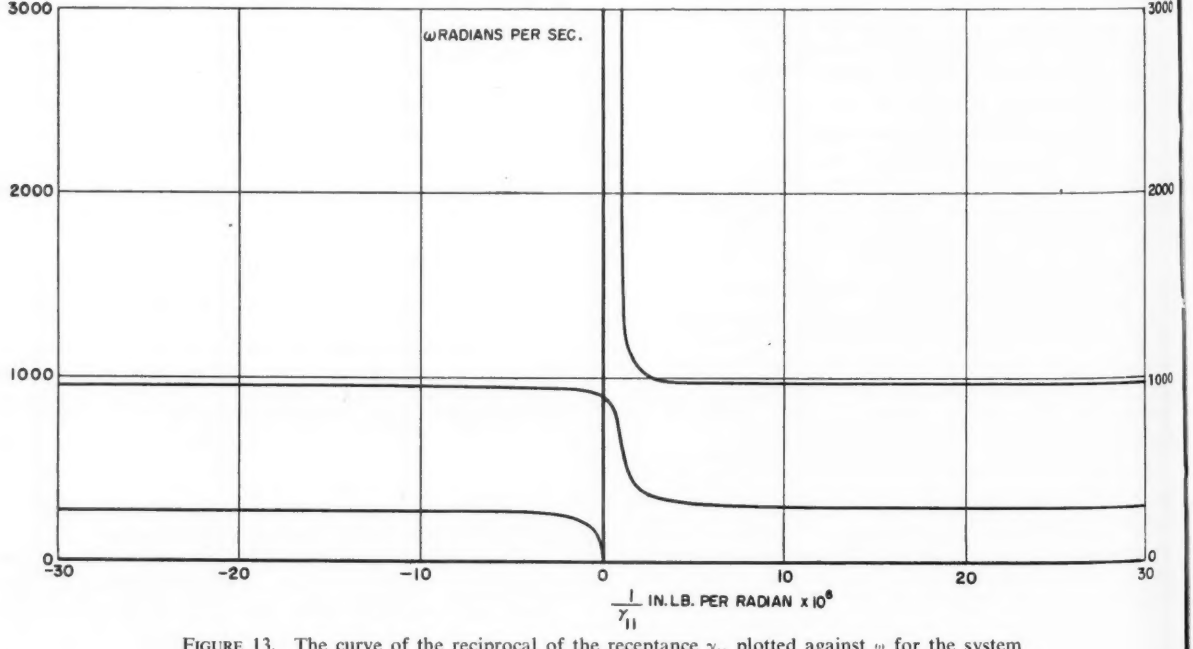


FIGURE 13. The curve of the reciprocal of the receptance γ_{11} plotted against ω for the system of Fig. 12.

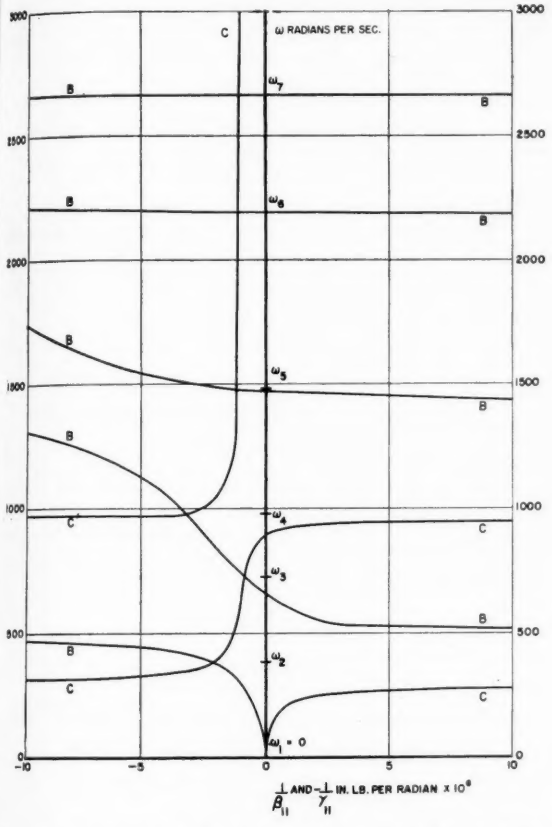


FIGURE 14. Curves B of $1/\beta_{11}$ against ω reproduced from Fig. 11 and curves C of $-1/\gamma_{11}$ against ω taken from Fig. 13. The natural frequencies of the composite system of B and C (Fig. 10(*b*)) are ω_1, ω_2 . . . . ω_7 given by the intersections of these curves.

This form of analysis suggests that the properties of simple sub-systems might be tabulated. It is only necessary that their receptance functions should be listed and such a catalogue is given in Table I. This table is concerned with simple torsional systems and the results apply equally to the axial motions of spring-connected masses. Comparable tables can readily be drawn up and used for other types of system.

A detailed review has been given of the principles involved in breaking a system down into components. Briefly, it is necessary to ensure that the component sub-systems fit together both as regards the forces and the displacements at their junctions. Table II contains a list of receptances of composite systems expressed in terms of the receptances of their components. The latter may be found, in practice, by the use of a catalogue such as Table I, by experiment or by calculation. Again, Table II may be readily extended.

Two methods exist for finding the natural frequencies of a given system and both are concerned with the condition under which the receptances are infinite. It is often convenient to find the frequency equation of a compound system in terms of the receptances of its components. A list of such equations is given in Table III. It is pointed out that, in one special (and easily identified) case, the equations in this table will not furnish a complete set of natural frequencies; the missing roots are easily found by investigating the sub-systems.

It has been shown that knowledge of the receptances of a system allow the calculation of the principal mode appropriate to any known natural frequency. These receptances may either be tabulated functions such as appear in Table I or they may be known only through those of conveniently chosen sub-systems. Whereas this method may be cumbersome for calculating the principal modes of a complex system, it may save much labour in finding the ratio of the displacements at two co-ordinates during free vibration; this is because it is not first necessary to calculate the relevant principal mode.

Finally, by means of an illustrative example, it is shown that the manufacturers of machinery would save both themselves and their customers much work if they were to publish details of their products in the form of curves. A convenient form of these curves is explained.

ACKNOWLEDGMENT

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APPENDIX I

THE RECIPROCAL PROPERTY $\alpha_{rs} = \alpha_{sr}$

When the system of Fig. 3 was being examined, it was shown that $\alpha_{12} = \alpha_{21}$. This result was found by direct substitution into the equations of motion and this procedure can be used to verify that the cross receptances between any pair of co-ordinates of any system are equal. However, it is clearly preferable to establish the relation

$$\alpha_{rs} = \alpha_{sr} \quad . \quad . \quad . \quad (52)$$

as a general theorem of linear oscillatory systems. The existence of this reciprocal property has been recognised for many years and a proof of it is given by Rayleigh[2];

it is dealt with in terms of receptances in Duncan's monograph[3]. But, since all the proofs with which the author is acquainted are framed in terms of the Lagrangian method, a treatment will now be given which is, perhaps, more elementary.

It is shown in books on elasticity (*e.g. see* Southwell[7]) that, when a system which obeys Hooke's law is loaded statically, the applied forces and couples $P_1, P_2 \ldots . P_m$ are related to the *corresponding* displacements* and rotations $\delta_1, \delta_2 \ldots . \delta_m$ through equations of the form

$$\left.\begin{aligned}\delta_1 &= f_{11}P_1 + f_{12}P_2 + \ldots\ldots\ldots\ldots + f_{1m}P_m \\ \delta_2 &= f_{21}P_1 + f_{22}P_2 + \ldots\ldots\ldots\ldots + f_{2m}P_m \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \delta_m &= f_{m1}P_1 + f_{m2}P_2 + \ldots\ldots\ldots\ldots + f_{mm}P_m\end{aligned}\right\} \quad . \quad (53)$$

where f_{11}, f_{12}, f_{21}, and so on are constants and are called "flexibilities" or "influence coefficients of displacement." Maxwell's well-known reciprocal relations are now derived by simple energy considerations and, according to these,

$$f_{rs} = f_{sr} \quad . \quad . \quad . \quad . \quad (54)$$

Now equations (53) and (54) apply, not only to an elastic body which is loaded statically (or very slowly), but they also hold good *at every instant* to a massless elastic body with time-dependent loading. However, they will not cover the general forced motion of a body that is endowed with mass because of the existence then of inertia forces.

For the sake of explanation, it is expedient to refer to a specific system—say a set of n rigid discs which are connected in some way by light elastic shafts so as to be capable of performing torsional oscillations. The rotations $x_1, x_2 \ldots . x_n$ of the discs from their mean positions may be taken as co-ordinates which "correspond to" torques applied to the discs. Suppose that time-dependent applied torques $F_1, F_2 \ldots . F_n$ act at $x_1, x_2 \ldots . x_n$ so as to throw the system into motion; then according to (53), the various displacements will be given by

$$\left.\begin{aligned}x_1 &= f_{11}(F_1 + F_1') + f_{12}(F_2 + F_2') + \ldots . + f_{1n}(F_n + F_n') \\ x_2 &= f_{21}(F_1 + F_1') + f_{22}(F_2 + F_2') + \ldots . + f_{2n}(F_n + F_n') \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n &= f_{n1}(F_1 + F_1') + f_{n2}(F_2 + F_2') + \ldots . + f_{nn}(F_n + F_n')\end{aligned}\right\} \quad (55)$$

where F_1', $F_2' \ldots . F_n'$ are the inertia torques at $x_1, x_2 \ldots . x_n$. Nothing has been done to invalidate the reciprocity relations (54) which still hold good.

When the applied torques are harmonic of the form $F_1 \sin \omega t$, $F_2 \sin \omega t, \ldots . F_n \sin \omega t$, a harmonic response of frequency ω is stimulated at $x_1, x_2 \ldots . x_n$. Thus,

$$\left.\begin{aligned}F_1' &= -I_1\ddot{x}_1 = I_1\omega^2 x_1 \\ F_2' &= -I_2\ddot{x}_2 = I_2\omega^2 x_2 \\ &\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots \\ F_n' &= -I_n\ddot{x}_n = I_n\omega^2 x_n\end{aligned}\right\} \quad . \quad . \quad . \quad (56)$$

*By this it is meant that $\delta_1, \delta_2 \ldots . \delta_m$ are measured "in the direction of" $P_1, P_2 \ldots . P_m$. Thus, if P_1 is a force, δ_1 is the displacement of its point of application in the direction of P_1 and if P_1 is a couple, δ_1 is the rotation of its point of application about the axis of P_1.

so that the inertia torques are proportional to the appropriate displacements. This feature permits the removal of the inertia torques from (55), because a modified version of the initial system can be imagined which will have the same motion but which will give rise to no inertia torques. This is arrived at by replacing the discs with light torsion springs having stiffnesses $-I_1\omega^2$, $-I_2\omega^2, \ldots . -I_n\omega^2$; by hypothesis, these negative spring stiffnesses do not render the motion unstable.

The displacements of this modified system are given by

$$\left.\begin{aligned}x_1 &= f_{11}'F_1 \sin \omega t + f_{12}'F_2 \sin \omega t + \ldots . + f_{1n}'F_n \sin \omega t \\ x_2 &= f_{21}'F_1 \sin \omega t + f_{22}'F_2 \sin \omega t + \ldots . + f_{2n}'F_n \sin \omega t \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n &= f_{n1}'F_1 \sin \omega t + f_{n2}'F_2 \sin \omega t + \ldots . + f_{nn}'F_n \sin \omega t\end{aligned}\right\} \quad (57)$$

where the new flexibilities f_{11}', f_{12}', f_{21}', and so on, still conform to the reciprocity relations (54), since the system is now devoid of mass. However, comparison of (57) with equation (19) reveals that the new flexibilities are the receptances of the original system, so that the general relation (52) is established.

APPENDIX II

The Use of Holzer's Analysis for the Determination of Receptances of Torsional Systems

Holzer's well-known method of analysis can be regarded as a means of calculating receptances for torsional systems of rigid discs connected by light shafts. This fact will now be illustrated.

Figure 15 shows a system of inertias I_a, $I_b, \ldots .$, joined by shafts whose torsional stiffnesses are k_a, $k_b \ldots .$ If the disc I_a has a displacement

$$x_a = X_a \sin \omega t \quad . \quad . \quad . \quad (58)$$

then the torque which must be applied to it (given by $I_a\ddot{x}_a$) is

$$-X_a I_a \omega^2 \sin \omega t = F_a \sin \omega t \quad \text{(say)} \quad . \quad (59)$$

This must be transmitted by the shaft k_a which is therefore

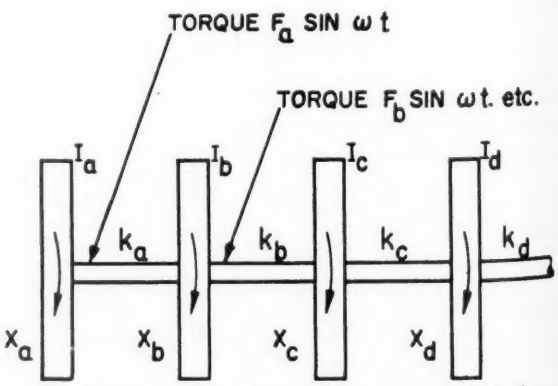

FIGURE 15. A typical "torsional system" of rigid discs coupled by light shafts.

TABLE IV

$\omega = 500$ rad./sec.; $\omega^2 = 0{\cdot}250 \times 10^5$/sec.^{2}

	I	$I\omega^2 \times 10^{-6}$	X	$XI\omega^2 \times 10^{-6}$	$-F \times 10^{-6} = \Sigma XI\omega^2 \times 10^{-6}$	$k \times 10^{-6}$	$-\frac{F}{k} = \frac{1}{k}\Sigma XI\omega^2$
Disc I_5	1	0·250	1	0·250	0·250		
Shaft k_4						2	0·125
Disc I_4	1	0·250	0·875	0·219	0·469		
Shaft k_3						2	0·234
Disc I_3	1	0·250	0·641	0·160	0·629		
Shaft k_2						2	0·314
Disc I_2	1	0·250	0·327	0·082	0·711		
Shaft k_1						2	0·355
Disc I_1	4	1·000	−0·028	−0·028	0·683		

Direct receptance $\beta_{11} = \dfrac{X_1}{F_1} = \dfrac{-0{\cdot}028}{-0{\cdot}683 \times 10^5} = 0{\cdot}041 \times 10^{-6}$ rad./(in. lb.).

[The reciprocal of this value ($= 24{\cdot}4 \times 10^5$ in. lb./rad.) lies on the curve shown in Fig. 11.]

TABLE V

$\omega = 500$ rad./sec.; $\omega^2 = 0{\cdot}250 \times 10^6$/sec.^{2}

	I	$I\omega^2 \times 10^{-6}$	X	$XI\omega^2 \times 10^{-6}$	$-F \times 10^{-6} = \Sigma XI\omega^2 \times 10^{-6}$	$k \times 10^{-6}$	$-\frac{F}{k} = \frac{1}{k}\Sigma XI\omega^2$
Disc I_7	5	1·250	1	1·250	1·250		
Shaft k_7						2	0·625
Disc I_6	5	1·250	0·375	0·469	1·719		
Shaft k_6						1	1·719
			−1·344		1·719		

Direct receptance $\gamma_{11} = \dfrac{X_1}{F_1} = \dfrac{-1{\cdot}344}{-1{\cdot}719 \times 10^5} = 0{\cdot}782 \times 10^{-6}$ rad./(in. lb.).

[The reciprocal of this value ($= 1{\cdot}28 \times 10^5$ in. lb./rad.) lies on the curve shown in Fig. 13.]

twisted through the angle $(F_a \sin \omega t)/k_a$. It follows that the displacement of I_b is given by

$$x_b = X_b \sin \omega t = \left(X_a + \frac{F_a}{k_a}\right) \sin \omega t \quad . \quad (60)$$

Thus, X_b can be calculated in terms of the amplitude X_a.

To produce the motion (60), the total torque on I_b (which is given by $I_b \ddot{x}_b$) is $-X_b I_b \omega^2 \sin \omega t$. The torque transmitted by the shaft k_b is therefore

$$(F_a - X_b I_b \omega^2) \sin \omega t = -(X_a I_a \omega^2 + X_b I_b \omega^2) \sin \omega t$$
$$= F_b \sin \omega t \quad \text{(say)} \quad . \quad . \quad (61)$$

That is, allowing for the twist of the shaft k_b, the displacement of I_c is

$$x_c = X_c \sin \omega t = \left(X_b + \frac{F_b}{k_b}\right) \sin \omega t \quad . \quad (62)$$

The amplitude X_c is thus known in terms of X_b which, in turn, can be found from X_a.

Attention can now be turned to the torque in the shaft k_c (of amplitude F_c) and thence the displacement x_d may be found; the passage from x_c to x_d in the calculation being the same as that from x_b to x_c. In this way, one may find the displacement x_n of the n^{th} disc I_n and the torque $F_n \sin \omega t$ which must be applied to I_n through k_n. Now, if there are $n-1$ shafts, so that there is no shaft k_n, then this torque must be applied by an external agency and the direct receptance at x_n is

$$\alpha_{nn} = \frac{X_n}{F_n} \quad . \quad . \quad . \quad . \quad . \quad (63)$$

Cross receptances α_{an}, α_{bn}, ... and so on, can be found in the same way.

Both X_n and F_n in equation (63) depend upon ω and the constants of the system; they are also both proportional to X_a so that the magnitude selected in the first place for this quantity can be conveniently taken as unity. The calculations are best done in tabular form and, in doing this, it is convenient to omit the multiplier $\sin \omega t$ and to work in terms of amplitudes only.

To illustrate this procedure, the calculation of the receptance β_{11} of the system shown in Fig. 10(*a*) is shown in Table IV for the frequency $\omega = 500$ radians/sec. (or $\omega^2 = 0{\cdot}25 \times 10^6$).

The method of Holzer can be used to determine the receptance at the free end of a shaft (as at the co-ordinate x_1 of the system of Fig. 12). Table V shows the calculation for γ_{11} for that system although, in practice, it is quicker to calculate such a simple receptance directly from the appropriate expression in Table I.

By the usual methods employed in Holzer's analysis, it is possible to find receptances of branched and geared systems.

TECHNICAL NOTES

Contributions to this Section of the JOURNAL *will be eligible for Journal Premium Awards and will normally be published within two months of being received.*

The Level of Safety Achieved by Periodic Inspection for Fatigue Cracks

by

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IT IS CURRENTLY BELIEVED in some parts of the world that the problem of structural fatigue in the case of aircraft having a modern multiple spar wing structure can be safely dealt with by allowing fatigue cracks to develop in service. The typical multiple spar wing has its bending tension area distributed between the "caps" of two or more spars and a large number of independent heavy stringers. It is believed in some quarters that complete failure in flight of any one spar boom or stringer due to fatigue will leave the wing with sufficient strength for the aircraft to land safely. This note does not discuss the validity of this belief, nor the level of safety so achieved, but attempts an objective assessment of a problem that arises as a consequence of this belief.

NOTATION

I_s Maximum acceptable structural incident rate, based on the flight as the unit of risk

H Total hours flown by any aircraft of the type (equivalent hours)

H_m Mean total hours for occurrence of detectable fatigue crack (equivalent hours)

h Representative flight duration (equivalent hours)

t Interval between inspections of suspect area (equivalent hours)

P_H Probability of a detectable fatigue crack developing in the hour of operation from H to $H+1$, in a randomly selected aircraft

P_H' Probability of a detectable fatigue crack developing in the hour of operation from H to $H+1$ in an aircraft in which there was no detectable crack at the last inspection

$$P_H' \simeq \frac{P_H}{\left(1-\sum_0^H P_H\right)}$$

P_n Probability per hour of the aircraft encountering a gust that will load the wing to "n" g or more

n Total wing load factor needed to cause a structural incident in an aircraft in which a detectable crack has developed ("g").

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1. THE PROBLEM

The practice of allowing fatigue weaknesses to reveal themselves in service has led on several occasions to the following sequence of events. No doubt this sequence will continue to recur:—

(1) A fatigue weakness reveals itself, either during inspection or in flight, in one aircraft of a type.

(2) An immediate inspection is made of all other aircraft of the same type at the particular point of weakness revealed by the first-detected failure.

(3) This immediate inspection reveals that a number (perhaps six or so) of aircraft of the type have small fatigue cracks at the same point.

(4) The manufacturer of the type develops a modification to the particular area to remove the fatigue weakness and supplies modification kits to the operators. This may take 6 to 12 months.

(5) In the meantime all uncracked aircraft continue to operate, safety being ensured by periodic *in situ* inspections of the revealed weak spot.

This note deals with the problem of setting the interval between the inspections under the 5th phase of this sequence so that an acceptable minimum level of safety is maintained. It is assumed that the first three phases have already occurred, and that information is available on the total flying hours of the aircraft concerned and the exact location and extent of the service cracks.

2. THE ACCEPTABLE LEVEL OF SAFETY

Use is made of the work done by the International Civil Aviation Organization on the "rational performance code." This code takes as its unit of risk the flight of an aircraft when its weight is up to the limit set by the performance code. The code aims to keep the risk of a "performance incident" on all such flights down to about 4×10^{-6}.

The "performance incident" is defined as occurring whenever the performance falls below a given datum. The datum is fixed at the minimum necessary for the pilot to land under full control at an appropriate aerodrome. A proportion of incidents, though by no means all, will lead to forced landings away from a suitable aerodrome, and a proportion of such landings will lead to a loss of life.

The corresponding incident in the structural realm may be termed the "structural incident." This is here defined as occurring whenever a small (but detectable)

fatigue crack spreads a considerable way through the wing structure under a dynamic flight load. A proportion of structural incidents will lead either immediately, or before the aircraft lands, to total wing failure and total loss of life.

It is considered that a structural incident is substantially more serious than a performance incident from the point of view of the safety of persons in the aircraft. Also practical variations in all-up weights have only a small effect on the danger of a structural incident, but have a great effect on the danger of a performance incident. That is, the structural incident rate on a unit flight basis is more fully exploited in actual operations than is the performance incident rate. For these reasons it is considered that the maximum tolerable structural incident rate is at least 10:1 less than the tolerable performance incident rate. The maximum acceptable structural incident rate, I_s, is therefore fixed at 4×10^{-7}, based on the flight as a unit of risk,

i.e. $$I_s \leqslant 4\times10^{-7}$$

is the condition of acceptable safety.

3. THE "DETECTABLE CRACK"

It is now convenient to introduce the concept of the "detectable crack." This is defined as the minimum size of fatigue crack that can be detected with certainty in an *in situ* inspection of the suspect area, using the proposed inspection method—usually a dye-penetrant method.

In general, this is a fairly vague concept. However, in particular cases it is not difficult to form a sufficiently accurate idea of what it means.

The detectable crack is generally much bigger than anyone familiar with the laboratory use of special inspection methods would imagine. Fatigue weak spots are commonly very inaccessible, they usually occur in areas heavily congested with rivets or bolts, and they are nearly always under a compressive stress when the aircraft is being inspected on the ground. Experience has shown that the minimum detectable crack usually has a length of $\frac{1}{4}$ in. or more on an accessible surface, and is of substantial depth.

4. THE PROBABILITY OF A DETECTABLE CRACK OCCURRING

The data obtained in the first three phases of the sequence described in the introduction enables a fair estimate to be made of the mean total flying hours H_m at which a detectable crack will occur in aircraft of the type.

The probability, P_H, of a detectable crack developing in the particular location in question in any individual aircraft in the flying hour from a total of H to a total of $H+1$ can be calculated from an assumed normal distribution, if the standard deviation of the distribution of occurrance of cracks is known.

In the present note the results are worked out for the alternative assumptions that the probability of cracking is normal with respect to H as argument, and with log H as argument. The present evidence is that the latter assumption is the more correct. To establish the standard deviation, a further point of the distribution curve is needed. The service cracking data available at this stage is never sufficient to enable an accurate estimate of standard deviation to be made by direct methods. However, a further point on the distribution curve is provided by the fact that the total risk of fatigue cracks occurring below half the mean life is generally believed to be acceptably low.

Assuming this belief to be well founded, on the present criterion of acceptable risk, this gives the condition

$$\sum_{0}^{\frac{1}{2}H_m} P_H \leqslant 4\times10^{-7}$$

Taking the equality, this relation is sufficient to establish the standard deviation, and so enable P_H to be calculated for every value of H. The results obtained are not very sensitive to the assumed value of this sum, provided it is small.

5. THE GUST LOADING NEEDED TO CAUSE A STRUCTURAL INCIDENT

When an aircraft in which a detectable fatigue crack has developed strikes a sufficiently large gust, a major failure of the structure will result—the extent of the failure depending on the strength and duration of the gust load and the nature of the affected structure. The minimum gust loading (expressed in terms of the total "g" loading in the wing) needed to cause such a gross development of a crack is denoted by n.

There is an important practical case which has given a definite lead on the estimation of the value of n. This is the well-known case of a failure that occurred a few years ago to an aircraft which had a three spar wing of typical modern design. The aircraft suffered a wing failure which spread through the front spar and the lower wing skin and stringers back to the centre spar, under a gust which imposed a measured load of $2g$. This failure started from a fatigue crack in the front spar cap —a crack which had a cross-sectional area of less than one per cent. of the total wing tension area.

Subsequent tests showed that the crack caused a static stress concentration sufficient to cause failure of the front spar cap under the stresses normally occurring under the measured $2g$ load. The failure of the front spar cap, by causing a spread of a stress concentration via the skin, and by the dynamics of stress transfer under a live load, initiated a progressive failure of the skin and stringers back to the centre spar. This progressive failure could not be accounted for by any simple static stressing analysis.

The lesson to be learnt from this failure is that when the initial fatigue crack in question occurs in a major component such as a spar cap or heavy stringer, the value of n should be taken at the "g" load at which the individual cracked component will fail. In practical cases this means that "n" may be as low as 40 per cent. of the actual ultimate load factor for the virgin wing.

6. THE PROBABILITY OF A GUST LOAD OF n OR GREATER OCCURRING

Once n is established, the probability, P_n, of a gust load of n or greater occurring in any hour's flying can be calculated.

This calculation is based on the aerodynamic characteristics of the type, and representative cruising speeds, heights and weights. Use is made of the available data on atmospheric gusts. This data is now fairly extensive and is expanding rapidly. For any given type and class of operation, quite accurate estimates of P_n are now possible. P_n is of course independent of H.

7. EQUIVALENT HOURS

It is known that the number of gusts encountered by a particular type of aircraft varies with the geographical area in which it operates, and also with the type of operation. However, the evidence is that the relative frequency of gusts of various magnitudes is fairly constant, and that it is the absolute frequency which varies from place to place, and from operation to operation. This enables hours flown under varying conditions to be reduced to a common basis—"equivalent hours"—by multiplying by a suitable factor.

In this note it is assumed that the hours flown by the various aircraft concerned are so reduced to a common basis. Similarly, the results obtained are in equivalent hours, and may in some cases have to be factored to give the real flying hours.

8. CALCULATION OF THE RISK OF A STRUCTURAL INCIDENT

A structural incident will occur when an aircraft in which a detectable crack has developed strikes a gust which loads the wing to $n\ g$ or more. The risk of this occurring during a flight of duration h hours in an aircraft that is being inspected at intervals of t hours is to be calculated.

P_H, of course, varies with H. For simplicity P_H can be taken as constant over the interval between inspections, at its mean value. This is a reasonable approximation where t is small compared with H_m and the standard deviation of the distribution.

The probability per hour P_H of a detectable crack occurring during the hour H to $H+1$ calculated in the ordinary way from the area under the probability curve between these limits is applicable only to an aircraft selected at random. In the present calculation the probability is conditioned in the case of the particular aircraft being considered, by the knowledge that at the last inspection no detectable crack was present. If the inspection periods are not too long, the conditioned probability P_H' is:—

$$P_H' \simeq \frac{P_H}{\left(1-\sum_0^H P_H\right)}.$$

The usual case is considered, where there are two symmetrical weak spots in each aircraft. A factor of 2:1 is used here to allow for the two symmetrical weak spots. This assumes that the failures of the two points in the one aircraft are random and independent. This is tantamount to the assumption that the variations of operational environment and gust loading actually encountered in service in the same number of equivalent flying hours do not cause appreciable variations in life, and that the scatter of failures is entirely due to mechanical and metallurgical differences between individual weak spots. If the reverse were true and the practical scatter were entirely due to environmental factors and actual gust loadings, then the factor would be 1:1. The true factor lies between 1 and 2. The wide scatter in laboratory tests, under identical environment and loading, leads to the belief that the first assumption is nearly correct, and the true factor is close to 2:1.

The risk of a structural incident is highest in the last flight of an inspection period, and this flight is therefore the critical one in establishing the level of safety.

The risk of a structural incident due to a detectable crack present at the beginning of this flight is:—

$$2P_nhP_H'(t-h).$$

The risk of a structural incident due to a detectable crack developing *during* the flight is:—

$$\tfrac{1}{2}\times 2P_nhP_H'h$$

(Remembering that the "n" gust load has to occur after the development of the crack.)

The total risk during this flight is therefore

$$I_s=2P_nP_H'h(t-\tfrac{1}{2}h). \quad . \quad . \quad (1)$$

The condition of minimum acceptable safety is thus:—

$$I_s=2P_nP_H'h(t-\tfrac{1}{2}h)\leqslant 4\times 10^{-7} \quad . \quad (2)$$

i.e.

$$t_{\max}=\frac{4\times 10^{-7}}{2P_nP_H'h}+\tfrac{1}{2}h. \quad . \quad . \quad (3)$$

While equation (3) continues to give a mathematical solution to the problem when t decreases to less than h, the solution then ceases to have a practical meaning.

When the aircraft reaches the total flying hours at which t decreases to equality with h, adequate safety can only be ensured by inspecting every flight, *and* simultaneously reducing the maximum permitted flight duration. When this occurs we can obtain the permitted duration and inspection interval by putting $t=h$ in equation (2).

$$2P_nP_H't(t-\tfrac{1}{2}t)\leqslant 4\times 10^{-7}$$

i.e.

$$t_{\max}=\sqrt{\left(\frac{4\times 10^{-7}}{P_nP_H'}\right)}=h_{\max}. \quad . \quad (4)$$

9. PRINCIPAL LIMITATIONS

The present computation of the risk of an incident neglects the possibility of incidents arising from a series of gusts of intensity less than n, acting on a member in which a detectable crack has developed. For example, if $n=2g$ in a particular case, it is possible that three $1{\cdot}8g$ gusts would do as much total structural damage to a cracked wing as one $2g$ gust, and the former combination of gusts will occur more frequently than the latter single gust. However, generally it is considered unlikely that a series of moderate level gust loads could cause so serious a failure as that which can arise from a sudden failure of a major element under a single large gust. Ignoring this factor tends to make the present estimates of safe inspection intervals on the high side.

It has also been tacitly assumed that a crack will not propagate appreciably past the detectable level by the

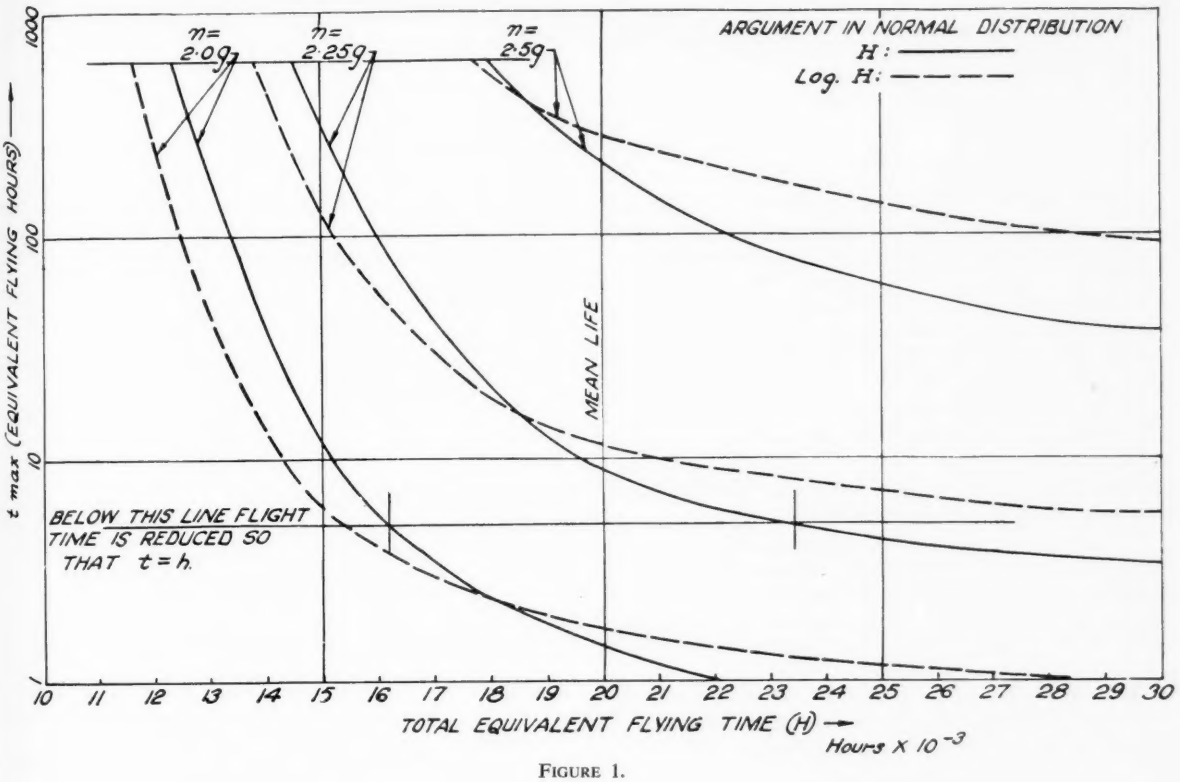
n= 2·09g
n= 2·25g
n= 2·5g
ARGUMENT IN NORMAL DISTRIBUTION
H: ——
Log. H: — — —
MEAN LIFE
BELOW THIS LINE FLIGHT TIME IS REDUCED SO THAT t = h.
t max (EQUIVALENT FLYING HOURS) →
1000
100
10
1
10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
TOTAL EQUIVALENT FLYING TIME (H) →
Hours X 10⁻³

FIGURE 1.

ordinary fatigue process under the alternating loads applied by the common, low level gusts in a time of the order of the inspection period.

There is some evidence that the propagation time in a laboratory test on a single solid specimen at stress levels equivalent to these frequent gusts is of the order of 7 per cent. of the mean life. This is the propagation time from the first appearance of a crack to static failure under the applied stresses.

It is considered that when the calculated inspection interval is less than one per cent. of the mean life, the assumption is reasonably valid. But as the calculated inspection interval increases, it will become increasingly in error due to this factor alone. For this reason it is considered that the safe inspection interval calculated by this method should be subject to an arbitrary upper limit of $2\frac{1}{2}$ to 3 per cent. of the mean life.

10. EXAMPLE

The following hypothetical example illustrates the method. The figures used are typical of a large modern piston-engined transport operating over international trunk routes:—

$H_m = 20{,}000$ hours.

$n = 2\,g$; $2{\cdot}25\,g$; and $2{\cdot}5\,g$.

$P_n = 5 \times 10^{-4}$; $1{\cdot}58 \times 10^{-5}$ and 5×10^{-7} respectively. (Based on a typical cruising speed and altitude.)

$h = 5$ hours.

Arbitrary upper limit to $t = 600$ hours.

Figure 1 gives plots of the maximum acceptable inspection interval per individual aircraft as a function of the total hours flown by the aircraft concerned for each of the three values of n and the two types of assumed normal argument.

11. CONCLUSIONS

Hitherto it has been the practice to set the inspection period t_{max} at a fixed number of hours for all aircraft of the type. This interval has been set by an intuitive process in which normal inspection periods and normal operational planning have played an excessively important part.

This note shows that to ensure that all operations are equally safe, the inspection interval should be varied with the total equivalent hours flown by each aircraft. In practical cases it may also show that for aircraft which have flown a high total number of equivalent hours, the acceptable inspection interval is less than the normal average flight duration. This means that inspecting the aircraft before every flight does not produce an acceptable level of safety unless flight durations are limited to the same figure as the permitted inspection interval. In some cases the permitted flight times will become so low that the aircraft is in effect grounded until modified.

ACKNOWLEDGMENTS

The author wishes to thank the Director-General of Civil Aviation of Australia for permission to publish this note. He also wishes to thank his colleagues, P. S. Langford, J. C. Thorpe and M. R. Weston, for their constant assistance in developing this approach to the problem.

Empirical Formulae for the Determination of Stress Curves

by

A. C. NICHOLLS

(*Structures Department, Bristol Aeroplane Co.*)

IN THE ANALYTICAL presentation of curves the main requirements perhaps are those of accuracy and simplicity and the problem confronting the engineer is the efficient combination of the two. A further important consideration is that of the range over which the empirical formula (or formulae in some cases) is to be effective, for clearly the greater this range is, the more useful will be the formula.

It is obviously desirable that any equation that is chosen should be as close a fit as possible to the practical curve. It must not be sensitive to small parametric differences and it should be possible to determine these parameters with a fair degree of precision.

The question of simplicity means that not only should the parameters be few in number but that their determination should be relatively straight-forward. A further desirable property of the equation would be that the process of differentiation and integration might be effected without much difficulty.

Since stress curves are of immediate interest perhaps the requirements could be expressed at this stage in more specific terms. An equation of the form[1]

$$\varepsilon = (f/E) + \phi(f)$$

where $\phi(f)$ is a function of the stress, is an obvious choice as this caters for both the elastic and plastic range with one formula. All that is required is that $\phi(f)$ should be accurate over the range $(0-f_5)$ and $(0-f_2)$, these being the appropriate stress ranges in tension and compression.

Two equations which are of the above form, both of which have only two parameters, are the following:

$$\varepsilon = \left(\frac{f}{E}\right) + K\left(\frac{f}{E}\right)^m \text{—Ramberg and Osgood.} \qquad (A)$$

$$\varepsilon = \left(\frac{f}{E}\right) + \beta\left[\cosh\frac{\alpha f}{E} - 1\right] \text{—Rao and Leggett.} \qquad (B)$$

These will be considered in turn in greater detail.

NOTATION

f and ε = stress and strain

f_{05}, f_1, f_2 and f_5 = 0·05, 0·1, 0·2 and 0·5 per cent proof stresses. (lb./in.²)

E, E_T and E_s = elastic, tangent and secant moduli. (lb./in.²)

f_T = stress corresponding to 0·14 strain. (lb./in.²)

RAMBERG AND OSGOOD

As stated by A. J. Barrett[2], this equation may be expressed more readily in the form $\varepsilon = (f/E) + B(f/f_2)^n$ for by satisfying the condition that this curve pass through f_2 (the 0·2 per cent. proof stress), at the appropriate strain means that the equation may further be simplified to read

$$\varepsilon = (f/E) + 0{\cdot}002\,(f/f_2)^n.$$

n may be obtained by making the curve pass through f_1, in which case

$$n = \frac{\log 2}{\log f_2 - \log f_1}$$

In order that this curve may pass through f_5, n must also be equated to

$$\frac{\log 2{\cdot}5}{\log f_5 - \log f_2},$$

and eliminating n, from both these equations yields the following result: $f_5/f_2 = (f_2/f_1)^{1{\cdot}322}$. This ensures that the curve will satisfy f_1, f_2 and f_5 at their respective strains. Where f_{05} is taken instead of f_5 a simpler condition is obtained—namely that $f_1^2 = f_2 \cdot f_{05}$.

By differentiation and rearrangement the following additional expressions are obtained:

$$\frac{f}{E_T} = \left(\frac{f}{E}\right) + n\phi(f)$$

$$\frac{E}{E_T} = 1 + \frac{nE}{f}\phi(f)$$

$$\frac{E}{E_s} = 1 + \frac{E}{f}\phi(f)$$

Where $\phi(f) = 0{\cdot}002\,(f/f_2)^n$.

A significant feature of this last equation is that it may be expressed in the linear logarithmic form $\log\phi(f) = A\log(f) + C$. This fact serves as a useful check on its accuracy.

Figure 1 reveals just how closely the theoretical curve approaches the test values.

RAO AND LEGGETT

$$\text{By equating } \alpha \text{ to } \frac{nE}{f_T} \text{ and } \beta \text{ to } \frac{\varepsilon_{max}}{\cosh n}$$

the expression becomes

$$\varepsilon = \left(\frac{f}{E}\right) + \varepsilon_{max}\left[\frac{\cosh nf/f_T}{\cosh n} - \frac{1}{\cosh n}\right]$$

and since n is invariably greater than 10, the last term may safely be omitted. This reduces the expression to the form used by the R.Ae.S.[3], which may further be simplified to read

$$\varepsilon = (f/E) + \varepsilon_{max}\,\varepsilon^{-n[1-f/f_T]}$$

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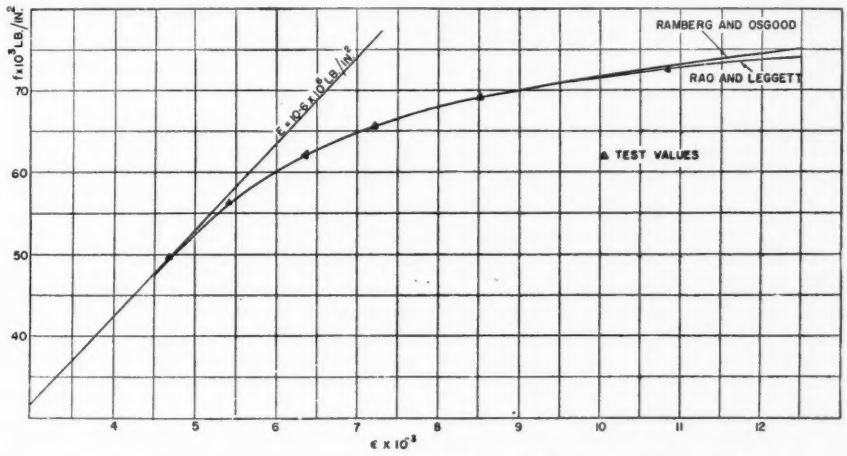

FIGURE 1. Stress-strain curve for Noral 75S (DTD.683).

This formula is practically identical and certainly far easier to handle.

By equating n to

$$\frac{\log_e [500\varepsilon_{max}]}{1-f_2/f_T}$$

the curve must pass through f_2 at the appropriate strain. The condition that it passes through f_1 at 0·001 strain also is obtained in the following manner:

$$0{\cdot}001 = \varepsilon_{max}\, e^{-n[1-f_1/f_T]}$$

$$n(1-f_1/f_T) = \log_e 1{,}000\varepsilon_{max}$$

Similarly

$$n(1-f_2/f_T) = \log_e 500\varepsilon_{max}$$

Equating ε_{max} to 0·14, as suggested by the R.Ae.S. and dividing one equation by the other reduces the above finally to the simple relationship

$$f_T = 6{\cdot}13(f_2-f_1)+f_2. \quad . \quad . \quad (1)$$

Now clearly f_T is not necessarily equal to the ultimate stress, but then neither is 0·14 the true value of the maximum strain. Furthermore, since the range of the curve that is of interest is from 0 to f_5 at the most, what happens at ε_{max} or f_T is somewhat irrelevant. ε_{max} in fact may safely be equated to a constant (*e.g.* unity) without affecting the essential properties of the curve, for

supposing $\varepsilon = (f/E) + Ae^{-m(1-f/f_t')}$

where $m = B/(1-f_2/f_T')$

Then $Ae^{-m(1-f/f_T')} \equiv \varepsilon_{max}\, e^{-n(1-f/f_T)}$

Therefore $\dfrac{m}{f_T'} = \dfrac{n}{f_T}$ *i.e.* $\dfrac{B}{(f_T'-f_2)} = \dfrac{\log_e 500\varepsilon_{max}}{(f_T-f_2)}$

and $Ae^{-m} = \varepsilon_{max}e^{-n}$.

The formula could further be simplified by equating ε to

$$f/E + e^{(\alpha f+\beta)}.$$

However, in view of the fact that the R.Ae.S. has used 0·14 for the constant A, this will be adhered to in order to facilitate direct comparison. The only change that will be made is that f_T will no longer be the ultimate stress but will be equated to $6{\cdot}13(f_2-f_1)+f_2$ for reasons stated earlier.

Like the Ramberg and Osgood formula, this curve may also be expressed linearly, for $\log\phi(f) = \alpha f + \beta$.

Being of an exponential form it is not surprising to find that differentiation of the formula is quite straightforward, and the following expressions may be obtained without much difficulty:

$$\frac{f}{E_T} = \left(\frac{f}{E}\right) + n\left(\frac{f}{f_T}\right)\phi(f).$$

$$\frac{E}{E_T} = 1 + \frac{nE}{f_T}\phi(f)$$

$$\frac{E}{E_s} = 1 + \frac{E}{f}\phi(f)$$

where $\phi(f) = 0{\cdot}14e^{-n[1-f/f_T]}$.

Having obtained the value of f_T which will guarantee the curve passing through f_1 and f_2, consider the condition to satisfy both f_2 and f_5.

$$n(1-f_2/f_T) = \log_e 70$$

$$n(1-f_5/f_T) = \log_e 28$$

from which is obtained

$$f_T = 4{\cdot}64(f_5-f_2)+f_2 \quad . \quad . \quad (2)$$

Equating (1) and (2) yields the following relationship:

$$\frac{f_5-f_2}{f_2-f_1} = 1{\cdot}32$$

which is the condition that the curve will pass through f_5, f_2 and f_1. Although this is not quite met in practice, very little adjustment is required in order that it may be satisfied. Using the specification figures quoted in AP.970 Vol. II, the necessary modification is about 200 lb./in.² for aluminium alloys and twice that for most steels. This seems reasonable considering that the specification figures are quoted to the nearest 1,000 lb./in.² in any case.

In the analysis of compression panels or struts where the ratio of f to E_T is required, it is essential that the original stress-strain curve be determined as accurately as possible. For this purpose the use of $f_{0\cdot5}$ in place of f_5 is to be preferred. It may be shown quite simply that

the condition for the curve to pass through f_{05}, f_1 and f_2 at the appropriate strains is that

$$\frac{f_2-f_1}{f_1-f_{05}}=1{\cdot}0.$$

A specimen calculation sheet (Fig. 4) follows from which may be obtained both accurately and rapidly the requisite stress curves. The work may further be reduced by making $E=10\times10^6$ lb./in.^{2} for aluminium alloys and 30×10^6 lb./in.^{2} for steels. This not only facilitates the presentation of the curves but also has little effect on their overall accuracy.

Figure 1 is a stress-strain curve based on certain test results[4]. This can be compared with the previous method.

Material

$f_1=$ lb./in.^{2} $\quad \varepsilon=f/E+\phi\,(f)$

$f_2=$ lb./in.^{2} $\quad f/E_T=f/E+n\,f/f_T\,\phi\,(f)$

f_{05} or $f_5=$ lb./in.^{2} $\quad E/E_T=1+\dfrac{nE}{f_T}\phi\,(f)$

$E=\times10^6$ lb./in.^{2} $\quad E/E_S=1+\dfrac{E}{f}\phi\,(f)$

(*a*) (*b*)

$\dfrac{f_5-f_2}{f_2-f_1}=$; $\simeq 1{\cdot}32$. $\dfrac{f_2-f_1}{f_1-f_{05}}=$; $\simeq 1{\cdot}0$

Let $f_2=$ lb./in.^{2}

$f_1=$ lb./in.^{2}

$f_T=6{\cdot}13\,(f_2-f_1)+f_2$

$=$ $=$ lb./in.^{2}

$n=4{\cdot}25\,f_T\div(f_T-f_2)$

$=$ $=$

$\dfrac{n}{f_T}E=$ $\quad \phi\,(f)=0{\cdot}14\,e^{-n\,[1-f/f_T]}$

$\phi\,(f)\times10^{-3}$	$n\,(1-f/f_T)$	$f\times10^3$ lb./in.^{2}	f/f_T $n\phi\,(f)\times10^{-3}$	$\varepsilon\times10^{-3}$	$f/E_T\times10^{-3}$	E/E_1	E/E_s	E_T/E	E_s/E
5	3·33								
2	4·25								
1	4·94								
0·5	5·64								
0·2	6·55								
0·05	7·25								
0·1	7·94								
0·02	8·85								
0·01	9·55								
0·005	10·24								

FIGURE 4. Stress-strain calculation sheet.

CONCLUSION

In their modified state both equations yield remarkably accurate curves. There seems little to choose between them from that point of view and while the first equation is perhaps easier to integrate the second one is probably quicker to evaluate.

It is hoped that in the future one or other of these empirical equations may be more widely applied.

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APPENDIX

ANALYSIS OF COMPRESSION TEST ON NORAL 75S (DTD.683) BAR

$f_2=69{,}300$ lb./in.^{2}

$f_1=65{,}600$ lb./in.^{2}

$f_{05}=62{,}300$ lb./in.^{2}

$E=10{\cdot}6\times10^6$ lb./in.^{2}

$f_4=72{,}600$ lb./in.^{2}

$f_{01}=56{,}300$ lb./in.^{2}

L.P. $=49{,}700$ lb./in.^{2}

(*a*) Using modified Ramberg-Osgood formula.

$$f_1^2/f_2\,.\,f_{05}=\frac{43{\cdot}0\times10^8}{43{\cdot}2\times10^8}\simeq1{\cdot}00$$

Let $f_2=69{,}300$ lb./in.^{2}

$f_1=65{,}600$ lb./in.^{2}

$$n\ \log\ f=\log\left[\frac{\phi\,(f)}{0{\cdot}002}\right]+n\ \log\ f_2$$

$$n=\log 2/\log\frac{69{,}300}{65{,}600}=\frac{0{\cdot}30103}{0{\cdot}02379}=\underline{12{\cdot}65}$$

$$n\times\log\ f_2=4{\cdot}8407\times12{\cdot}65=\underline{61{\cdot}24}$$

$\phi\,(f)\times10^{-3}$	$\log\dfrac{\phi\,(f)}{0{\cdot}002}$	$n\log f$	$f\times10^3$ lb./in.^{2}	$n\phi(f)\times10^{-3}$	E/E_T	E_T/E
5	0·4	61·64	74·7	—	8·47	0·118
2	0	61·24	69·3	25·3	4·65	0·215
1	−0·3	60·94	65·6	12·65	2·93	0·341
0·5	−0·6	60·64	62·2	6·33	2·02	0·495
0·2	−1·0	60·24	57·8	2·53	1·438	0·695
0·1	−1·3	59·94	54·7	1·265	1·232	0·812
0·05	−1·6	59·64	51·9	0·633	1·122	0·891
0·02	−2·0	59·24	48·2	0·253	1·053	0·950
0·01	−2·3	58·94	45·6	0·1265	1·028	0·973
0·005	−2·6	58·64	43·2	0·0633	1·015	0·985
4	+0·3	61·54	73·3	—	—	—

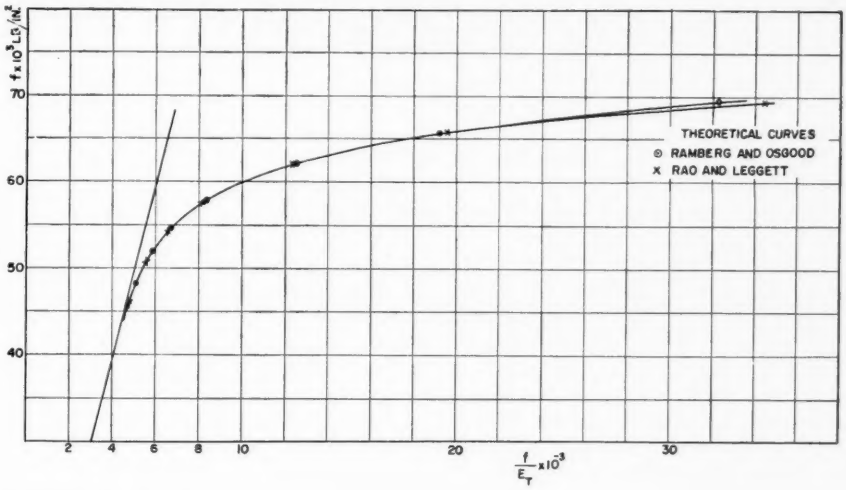

FIGURE 2. Stress-tangent modulus curve for Noral 75S.

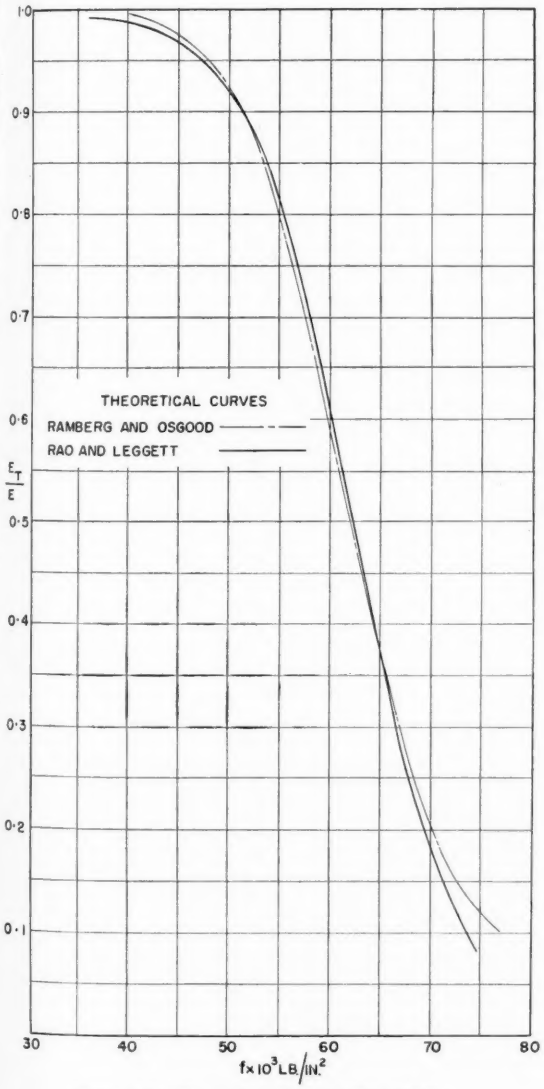

FIGURE 3. Tangent moduli for Noral 75S.

(*b*) Using modified Rao and Leggett formula:—

$$\frac{f_2 - f_1}{f_1 - f_{0.5}} = \frac{3{,}700}{3{,}300} \quad ; \quad \frac{3{,}500}{3{,}500} = 1{\cdot}00$$

Let $f_2 = 69{,}200$ lb./in.²

$f_1 = 65{,}700$ lb./in.²

$$f_T = 6{\cdot}13\,(f_2 - f_1) + f_2 = 21{,}500 + 69{,}200 = \underline{90{,}700 \text{ lb./in.}^2}$$

$$n = 4{\cdot}25\, f_T/(f_T - f_2) = \frac{4{\cdot}25 \times 90{,}700}{21{,}500} = \underline{17{\cdot}95}$$

$$\frac{nE}{f_T} = 1{\cdot}98 \times 10^3$$

$\phi(f) \times 10^{-3}$	$n\,(1 - f/f_T)$	$f \times 10^3$ lb./in.²	$\frac{f}{f_T}\, n\phi(f) \times 10^{-3}$	E/E_T	E_T/E
5	3·33	73·9	—	10·90	0·092
2	4·25	69·2	27·5	4·96	0·202
1	4·94	65·7	13·0	2·98	0·336
0·5	5·64	62·2	6·15	1·99	0·503
0·2	6·55	57·6	2·28	1·396	0·716
0·1	7·25	54·2	1·07	1·198	0·835
0·05	7·94	50·6	0·50	1·099	0·910
0·02	8·85	46·0	0·182	1·040	0·962
0·01	9·55	42·5	0·084	1·020	0·980
0·005	10·24	38·9	0·038	1·010	0·990
4	3·56	72·7	—	—	—

Definition of Specific Impulse

S. W. GREENWOOD, M.Eng., A.F.R.Ae.S.

REGARDING the recent Technical Note of Mr. Asquith and Mr. Whitaker (May 1954 JOURNAL):

1. The dependence of the weight of a given mass on location is a sufficient reason for the abolition of the "weight" definition of specific impulse. This is likely to become increasingly apparent as rockets penetrate further into interplanetary space.

2. It is convenient that the "weight" definition gives values of specific impulse which are independent of the units employed, provided that the units of thrust and weight are the same, for the unit of time employed in the definitions under discussion is the second. This raises the question of whether the unit of time should have been specified in the definitions. It seems to me that an improvement on fundamental grounds would be to define $(S.I.)_m$ as thrust per unit mass of propellants consumed per unit time and $(S.I.)_w$ as thrust per unit weight of propellants consumed per unit time.

3. I would emphasise that $(S.I.)_m$ does not vary with location. For example, when the units (lb. force × sec.)/(lb. mass) are employed, the lb. force in the numerator is a force of defined magnitude, and does not vary with location.

4. May I sum up by proposing that the definition of specific impulse given by thrust per unit mass of propellants consumed per unit time be adopted as standard, and that the units (lb. force × sec.)/(lb. mass) be employed when using the foot-pound-second system.

K. ASQUITH AND J. G. WHITAKER

WE GAVE TWO main reasons in favour of the "weight" definition—(1) it considerably reduces the possibility of confusion over units, (2) it "legalises" the almost universal practice of expressing S.I. in seconds. We do not consider that Mr. Greenwood's remarks invalidate our conclusion that the "weight" definition is preferable.

We agree with Mr. Greenwood that provided forces are everywhere measured in *earth* gravitational units, $(S.I.)_m$ is independent of location: this is because the universal use of earth gravitational units is tantamount to using absolute units. But if a true gravitational system of units is employed, $(S.I.)_m$ must vary with location.

F. W. THORNE
(*Royal Naval College*)

YOUR CONTRIBUTORS on Specific Impulse in the May JOURNAL say, "The mass definition, on the other hand, gives a value of specific impulse which varies according to the units used."

It must be realised that the value of a quantity *cannot* depend on the units chosen. To quote therefore the example on page 371 in gravitational units, namely:—

$$(S.I.)_m = \frac{2{,}000}{10} \frac{\text{lb. wt.}}{\text{lb. per sec.}}$$

if we observe that the lb. wt. or Lb. is related to the lb. mass or lb. by

$$\text{Lb.} = \frac{32{\cdot}2 \text{ lb. ft.}}{\text{sec.}^2} \times \text{substitute}$$

we have $(S.I.)_m = \frac{2{,}000}{10} \frac{\text{Lb.}}{\text{lb./sec.}} \left[\frac{32{\cdot}2 \text{ lb. ft.}}{\text{Lb. sec.}^2}\right]$

$$= 6{,}440 \text{ ft./sec.}$$

Whatever the merits or demerits of any system neither can alter the value of a given quantity.

K. ASQUITH AND J. G. WHITAKER

WE CAN AGREE with Professor Thorne that no physical quantity can be altered by varying the system of measurement, in the sense that the length of a ruler, for example, is the same whether its length be recorded in inches or centimetres. It is of course the numerical value of the length that varies with the system of measurement, and perhaps we should have used the words "numerical value" instead of "value" in the paragraph to which Professor Thorne refers.

We maintain our assertion, however, that in recording specific impulses, there is more likelihood of confusion as to numerical value, and as to the nature of the unit, if the mass definition is employed.

The Flexural Centre or Centre of Shear

by

J. A. JACOBS

PROFESSOR KOITER, in his note (January 1954 JOURNAL) states that I (April 1953 JOURNAL) was "apparently unaware" of Weber's two papers on the subject, published in 1924 and 1926. I was aware of the papers, having cited them in the bulletin[1] by Seely, Putnam and Schwalbe on "The Torsional Effect of Transverse Bending Loads on Channel Beams."

Some references in addition to those already cited may be of interest. The unorthodox behaviour of a channel beam in bending was observed and reported by C. Bach[2] in 1909-1910. A. and L. Föppl[3] wrote a section on the "Schubmittelpunkt" in their book "Drang und Zwang."

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THE LIBRARY
Reviews

HYDRAULIC SYSTEMS AND EQUIPMENT. *R. Hadekel. Cambridge University Press 1954. 224 pp. Illustrated. 18s.*

It has long been appreciated that while there are many excellent textbooks on the elements of fluid mechanics, there are few which deal, so to speak, with applied hydraulics. Mr. Hadekel's excellent book covers this branch of the art adequately and is one which designers, project engineers, and all who use hydraulic machines, will find of interest.

Following an introductory chapter in which the basic elements of a hydraulic circuit are defined and some useful formulæ are given in a convenient form, there are eight chapters dealing with circuit design. These form about two-thirds of the book and describe the problems encountered and the solutions usually adopted, in considerable detail.

The first part of these eight chapters is concerned with fundamental principles of circuit design, where the various basic components used in the circuit are introduced. It also deals with the problem of pump idling, open centre and constant pressure systems, and the methods of controlling all ranges of hydraulic horsepower. The second part of the chapters devoted to circuit design includes a full chapter of various sequencing circuits with a further three chapters dealing with stored energy, pumps and motors, and speed control. The remaining four chapters of the book are given to general component design considerations and deal with the question of seals, the design of jacks and accumulators, valve design, hydraulic piping, tanks and filters.

All chapters are profusely illustrated with excellent diagrams which are drawn in a clear and consistent manner.

The book contains some extremely useful material, particularly in that part devoted to circuit problems. The information contained in the chapter on sequencing circuits will be of value to the designer with a problem, as the basic principles of sequencing circuits are arranged so that the necessary information can easily be extracted and a circuit to meet a particular need readily designed. Any adverse criticism is only of a minor nature; for instance, the book would be improved if a fundamental problem such as sealing were dealt with in more detail and, in particular, if a little more space were devoted to the so-called "shear seal methods." Perhaps, also, at a later date, some further information could be added concerning instability of valves and pilot-operated components, and also on the subject of energy absorption by various means such as hydraulic jacks with built-in damping valves.

The book is to be highly recommended and should be on the book shelf of any engineer or student who wishes to keep abreast of this particular subject.—K. G. HANCOCK.

ELECTRONICS. *T. B. Brown. John Wiley, New York, 1954. 545 pp. Diagrams. 60s.*

This book has been written to be of use to Professor Brown's own students of physics and engineering at George Washington University, where a basic practical course is given in thermionic valve theory and circuitry. It aims at being a true student textbook, correlating the laboratory with the lecture room. The author has had over thirty-five years experience of teaching electronics and this book is the outcome of experiments tried out on classes to find suitable contents, the correct combination of practical work and theory, and the best order of presentation. In 1949 the text was mimeographed and modified by later experience to become published in its final form in 1954.

The book contains descriptions of laboratory and demonstration experiments as part of the text and a collection of representative problems at the end of each chapter. The scope is fairly wide from vacuum valves, gas-filled valves, photocells, modulation-amplitude and frequency, modern non-linear circuit techniques such as two state devices and digital computing, to high frequency work such as Klystrons, Grounded Grid amplifiers (but not distributed amplifiers). Good use is made of graphical analyses in preference to purely algebraic explanations and emphasis is continually made to the non-linear nature of most systems.

The principle of feedback is well done by starting with simple examples such as the feedback effect of a cathode resistor. In particular, the graphical method of dealing with the reduction of amplitude distortion by negative feedback clarifies the underlying physical mechanism.

Stability, however, is poorly treated. The Nyquist diagram is shown by name without explanation of any stability criterion in Sec. 6: 13 and refers back to Sec. 6:9 for unstable conditions. This latter, however, only mentions the possibility of continuous oscillation by positive feedback and refers forward to Sec. 6:23. Here loop gain is treated as a real positive quantity with oscillations ensuing for the loop gain equal to and greater than unity. The non-linear aspect with distorted sine-wave oscillation is well brought out. Nowhere, however, is the Nyquist diagram used and conditionally stable systems, so necessary for wide band amplifiers, are not even mentioned.

Curious too is the acceptance of vector ideas until well into the book. For instance, Sec. 6:12 deals with the inverse gain vector loci, with insets showing vector inversion, and yet in Chapter 10 simple vector diagrams are introduced and used to explain simple parallel and series resonance.

One of the six appendices deals with transistors and their circuitry and concludes with a small bibliography. Two appendices dealing with laboratory procedure and planning a laboratory are rather elementary and out of keeping with the rest of the book; "plate-current readings might be tabulated in a column headed plate-current, ma."

The book has a good index but no author index. There are very few references to original work even in modern developments, but the book is an excellent textbook for undergraduates who are actually undertaking a laboratory training course in electronics.—JOHN C. WEST.

THEORY AND DESIGN OF STEAM AND GAS TURBINES. *John F. Lee. McGraw-Hill, 1954. 502 pp. Illustrated. 64s. 6d.*

"Take a set of lecture notes, sprinkle liberally with line diagrams and manufacturer's photographs and serve cold with a garnishing of reprinted tables of data" might almost be said to be the standard American recipe for textbooks of the "student's introduction and practising engineer's handbook" variety and the work under review is no exception. One hastens to add that the present example

is quite a useful text for those requiring a fairly detailed survey of the field, though one is forced to add the usual complaints that the price is far too high for most private buyers, and that no answers are given to the examples.

Restricting attention to the gas turbine aspects as being of most interest to aeronautical engineers—and these occupy all but three of the sixteen chapters—one is led through chapters on Gas Turbine Types, Fundamentals of Thermodynamics, Gas Turbine Cycles and Elementary Gas Dynamics in a clear and conventional manner. The last of these includes a very brief survey of aerofoil theory. Incidentally, one example in this chapter (No. 6-4) concerning shocks in nozzles cannot be properly answered until the following chapter has been read. The latter covers the practical aspects of the Design of Nozzles, and presents a version of the familiar pressure-area plot (Fig. 7-12) for a convergent-divergent nozzle which is misleading: the flow after a normal shock is alleged to proceed along a constant energy (instead of constant entropy) line, a statement which, while true, implies to the uninitiated that there is an energy loss somewhere, presumably at the shock. It is also implied that the state point just after the shock is that of minimum area (*i.e.* sonic velocity) for the given entropy, which is not, in general, true.

There follow chapters on Energy Interchanges in Fluid Machinery, Design of Turbine Flow Passages, Mechanical Aspects of Turbine Design, Centrifugal Compressors and Axial Compressors. This latter is noteworthy for being based on British rather than American design data.

Chapters on Combustion and on the Regenerator are followed by a final one on the Gas Turbine Power Plant, covering performance aspects of the complete engine. (It is interesting that, while there is a complete chapter on steam turbine control and performance, gas turbine controls are not mentioned.) After a review of non-dimensional parameters, which includes the surprising statement that "Reynolds number is of little importance in high-speed turbo-machinery," the procedure for computing the overall performance from the component characteristics is dismissed in a few sentences: how few can be the authors of such books who have actually calculated the performance of an engine of any complexity! Typical component performance maps and running lines are illustrated, though Fig. 16-3, showing a compressor characteristic (as linked with its turbine) masquerading as that of a turbine, should be speedily replaced. The examples on this chapter, implying that aircraft gas turbines normally operate at the same non-dimensional condition irrespective of altitude, are dangerously misleading to a student working through without external aid. The volume ends with shortened data tables from the standard works of Keenan, Kaye and Keyes.

To summarise, the layout and content of this volume are reminiscent of various others, and the author's claim to originality in treating steam and gas turbines together can hardly be supported: as prescribed reading to accompany a lecture course it would certainly be useful, but pending revision in detail it should be treated with caution by the student reading unsupervised.—J. R. PALMER.

DIE EROBERUNG DES LUFTREICHES. *Peter Supf. Konradin Verlag-Stuttgart. 288 pp. Photographs. 22s. (In German.)*

In 278 packed pages, Mr. Supf has managed to give an admirably comprehensive review of the progress of human flight from its fantastic origins to a peep into the future.

Within the framework of his book, he has acted wisely in steering a central course, concentrating on the main issues of, and contributions to, aeronautical progress and keeping clear of by-lanes. This gives the reader a lucid picture of the major trends of development and spares him the confusion of digressions. No less was expected of a writer of Mr. Supf's experience and erudition; also it is refreshing to note that unlike his history of German aviation published in the Hitler days and bristling with pages of "steel and blood," the present work is tuned to a peaceful and world-embracing note. Apart from a chapter dealing with the military application of balloons and a brief review of the last war's developments, in which Coventry, Dresden and Hiroshima are mentioned in one breath, the author has abstained from expanding into the military field of aeronautical development.

It is however in the chapter on gliding that the author's love of flying comes to the fore; in quickly sketched pictures punctuated by first-hand reports, the entire panorama of the subtle art is revealed to the reader.

Here and there some inaccuracies have crept in, such as in the chapter on kites, where it is suggested that between the 17th century and 1916 nothing had been heard of kite flights; also, the discussion of the Wright brothers and their lawsuits leaves one with the impression that the author had perhaps had no access to all relevant sources.

Finally, the typography and illustrations are excellent and the binding designed by Heinz Renz is most attractive. The value of the book would be further enhanced by a bibliography. References to page numbers in the list of illustrations and the placing of the list of contents in front, instead of at the end of the book where it is at present, would contribute to an easier handling of future editions of this interesting book.—V. L. GRUBERG-GORE.

PROCEEDINGS OF THE SOCIETY FOR EXPERIMENTAL STRESS ANALYSIS—VOL. XI, No. 2. *Edited by C. V. Mahlmann and W. M. Murray. Published by the Society for Experimental Stress Analysis, Cambridge 39, Mass. 222 pp. Diagrams.*

Recent sets of Proceedings of the Society have been notable for the prominence given to papers dealing with the problem of fatigue, and the present volume is no exception. Apart from three individual papers on fatigue testing, there is a short symposium of five papers by recognised authorities on their subjects, the main theme of which may be gathered from the general title "The Interpretation and Application of Test Data." One of the three individual papers describes a 200,000 lb. testing machine in which the sequence of loads can be programmed beforehand in respect of both mean and cyclic loading, thus simulating field conditions as closely as possible. The accent in the two other papers, one of which is concerned primarily with plastic materials, is on "creep," and both describe apparatus in which several specimens can be tested simultaneously.

Although the five papers included in the symposium are all rather short, they are nevertheless of outstanding interest. The last of the five expounds the philosophy behind the general title very clearly. To put it simply, the problem faced is that of devising a set of tests that shall, without being impracticably long-winded, provide a reliable estimate of the efficiency of a proposed structural design after perhaps twenty or thirty years of continuous use.

A paper of special interest to aeronautical engineers deals with the theory and technique of measuring structural loads in flight. The only reliable way of doing this—by calibration of the structure in the laboratory for similarly applied loads—is beset with the difficulty that the flight

load distribution is unknown, and the value of the paper lies in the proposals it makes for overcoming this difficulty.

Among a number of papers on the subject of measuring stresses in the interior of a material, there is one that aims at drastically reducing the time that more orthodox methods of testing require. By removing successively-exposed surfaces of a sheet-metal specimen by chemical solution at the rate of 0·001 in. per minute, it is shown how, by continuously measuring the consequent change of curvature, a complete record of the residual stresses at varying depths from the sheet surface can be obtained in a fraction of the usual time.

The use of models for investigating dynamic loads is the subject of three papers. One of these discusses the reliability of plastics for use as material for model construction; another describes the use of small-scale models for measuring the dynamic performance of ship gun-turrets, while a third treats the problem of blast impact on buildings under various conditions.

There is an interesting paper on the use of vitreous enamels as a substitute for the more " temperamental " stress coat materials for the measurement of stresses at high temperatures. Another paper of topical aeronautical interest discusses the problem of extending to the three-dimensional case the use of the electrical-network analogue approach to the solution of structural problems.—D. WILLIAMS.

CHARACTERISTICS AND APPLICATIONS OF RESISTANCE STRAIN GAUGES. *National Bureau of Standards, 1954. 140 pp.*

This report on the symposium on Resistance Strain Gauges represents some of the important results of investigations by institutions from the United States, Holland, Switzerland and Sweden. Eleven papers are presented along with the transcriptions of the discussions that followed. Two aspects of measurements with resistance strain gauges are dealt with, namely the use of resistance strain gauges to the study of stress analysis and that in the field of instrumentation.

The first paper gives an account of some experimental determinations of Poisson Ratio for elastic and plastic strains on a specimen under tensile stresses of magnitude that do not involve necking. Although the specimen could not be considered ideally isotropic the authors report good agreement between observed and calculated values.

To the practical worker the cementing, water proofing and manipulation of bonded and unbonded type of resistance strain gauges is always of importance. Papers 2 and 3 deal with these practical aspects. A particularly interesting use of the latter type is illustrated in the design of an accelerometer. Two papers are devoted to the study of stresses in concrete structures. Certain fundamental aspects of the effect of shrinkage in " setting " in concrete blocks on the stress and strain with lapse of time are reported in paper 4. Although probably not intended to do so this is a paper describing mechanical adhesion. Paper 5 deals with the effect of rate of application of load on compressive strength as compared with static strength of concrete blocks.

A special type of resistance strain gauge free from transverse sensitivity is described by Gustafsson and Huggenberger in papers 6 and 7. Use of this gauge for the study of stress analysis of stressed skin structures should be of special interest to aeronautical engineers.

The three papers that follow deal with the varied applications of resistance strain gauges in the field of instrumentation. Examples are given of the following type of measurements. Loads in cables in tension in oil survey work, loads in paper manufacture, weighing of locomotives, aircraft and vehicles, fluid pressure measurements, torque meters and dynamometers.

The last paper deals with conducting films giving some account of the strain sensitivity and stability of films of metals and of colloidal graphite. Behaviour of graphite films in biaxial stress field is also discussed.

On the whole the book provides good reading to one interested in the applications of resistance strain gauges. The short accounts of the discussions that followed the reading of each paper are also very interesting. That no two research workers agree to or trust in the reproducibility and reliability of resistance gauges other than those that are familiar to themselves is amply illustrated in these discussions.—R. S. KHOT.

V2. *Major-General Walter Dornberger. Hurst and Blackett, London, 1954. 263 pp. 26 photographs. 16s.*

No matter how well or how badly a book bearing the title V2 by the former commanding officer of Peenemünde is written, it can hardly fail to be of absorbing interest to anyone in anyway connected with the art and science of aeronautics. There can be little doubt that this book will have a wider audience than this since it deals with a subject which brought itself to the attention of the general public in a most dramatic and unpleasant manner.

General Dornberger was in charge of liquid propellant rocket research and development for the German Army from 1930 to 1945 and his book is largely a semi-technical description of the work for which he was responsible, which culminated in the first successful experimental launching from Peenemünde in October 1942 of the V2, or A4 as the prototypes were designated. As this was a forerunner of the guided weapon on which so much of our technical effort is now being expended, albeit behind veils of security, the story of the early trials, and of the tribulations, will be of direct interest to many.

As a book, it suffers from many failings; it is perhaps out of place here to discuss the author's attitude to officialdom which provided the finance for his work or even to denationalisation but none the less these make an interesting comparison with that of Sir Frank Whittle recorded in his recently published book! Technically the book suffers from a lack of chronological order, resorting at times to the " flashback " so dearly loved by cinematographic workers, and also to a lack of detailed description where it would be of most value.

Regardless of any of its failings, this is a book of absorbing interest and should be read by many.—J. A. DUNSBY.

THE AMAZING MR. DOOLITTLE. *Quentin Reynolds. Cassell, 1954. 313 pp. 18s.*

I must admit that until I read this book I felt that the title was chosen to catch the eye. It was not until I had finished it that I realised that " amazing " is the *mot juste*. I doubt if any man alive today has smashed so many aeroplanes and continued to enjoy life—and in aeronautics to boot!

To most people, Doolittle means the famous first raid on Tokyo (and the full description of this raid and its sequels makes the book worthwhile) but this is not even the climax of a career that must have contributed a more steady and consistent stream of improvement to aeronautics than almost any other man's.

I cannot help feeling that had the book been an autobiography it might have been more readable since " Mr.

Shicklegrüber's" inventor tends to "lay it on with a trowel," but he has certainly written a detailed biography and it is pleasant to see due credit given to a great man's wife and their friends. Credit is also given to that petroleum company which is lucky enough to have Doolittle on the Board—so much, in fact, that I will not give it a further boost by mentioning it by name.

Those readers who gain the impression that General Doolittle was responsible for the entire softening-up of Germany that contributed to her defeat by the Americans, must remember that the book was written for American consumption about an American and by an American.—F.H.S.

MATERIALS AND PROCESSES. *James F. Young. Second Edition. John Wiley, New York. Chapman & Hall, London, 1954. 1,074 pp. Illustrated. 68s.*

The first edition of this work appeared in 1944. The present edition is considerably larger and most of the chapters have been completely re-written. In addition, there are new chapters on Metallographic Examination, Structure and Properties of Non-Metallic Materials, and on Statistical Methods used in Quality Control.

The book aims at presenting a broad study of materials and manufacturing processes from the viewpoint of the engineer and designer. It is in two parts, the first dealing with Materials and the second with Processes. The chapters in the first part dealing with the metallurgy of materials are particularly well written and contain much more detailed information than is normally found in books of this kind. The second part deals with casting processes, powder metallurgy, heat treatment, welding, manufacturing processes, and quality control.

The book is written, it would seem, mainly for the general engineer: the aeronautical engineer may well be disappointed that there is so very little on aluminium alloys, and even less on magnesium alloys. On the whole, however, the work is without doubt a good production and should prove very useful both as a reference book and as a textbook.—E.W.C.W.

TEXTBOOK OF PHYSICS. *Edited by R. Kronig. Pergamon Press, London, 1954. 855 pp. Diagrams. 70s.*

This is the first English edition of a Dutch text book which ran to three editions between 1946 and 1951.

The book is intended to give a survey of physics as a whole and the arrangements adopted was as follows: to present first the phenomenological parts of physics-mechanics, theory of vibrations, electrodynamics and optics; to pass from these to the theory of the atom and the atomic nucleus and to the atmospheric interpretation of the properties of matter in bulk; to devote two chapters to physical instruments, and to conclude with a survey of medical physics.

The arrangement and presentation are good, some of the more specialised or mathematical parts being set in smaller type, and there are twenty pages of biographical notes and an excellent index.—E.C.P.

FUNFSTELLIGE TAFEL DER TRIGONOMETRISCHEN FUNKTIONEN. (*Five-figure tables of trigonometric functions.*) (*English Preface.*) *B. Sticker. Dümmlers, Bonn, 1954. 52 pp. DM.7.60.*

The book contains the standard five-figure tables of sin, cosec, tan, cotan, cos, sec, functions. The angle increment is in single minutes, with tables for linear interpolation in decimals of a minute. A separate table provides for change from decimals of a minute to seconds and vice versa. A useful addition in the 0°-2° range is a table of functions for the calculation of cosec, sec and cot, tan functions which do not obey the linear interpolation rule in that range.—J.S.

HIGHER TRANSCENDENTAL FUNCTIONS, VOLS. I AND II. *Based, in part, on notes left by Professor Harry Bateman and compiled by the Staff of the Bateman Manuscript Project. McGraw-Hill, New York, 1953. 302+396 pp.*

The late Professor Bateman planned to include in his "Guide to the Functions" a detailed account of the properties of the most important functions and, as well, the historic origin and definition of, the basic formulæ relating to, and a bibliography for, *all* special functions ever invented or investigated. After his death, the California Institute of Technology and the Office of Naval Research combined to produce these two volumes, restricted to an account of the principal properties of the more important special functions. The books, which have been produced by the varitype process, cover the Gamma function, the hypergeometric function and generalisations, Legendre functions, the confluent hypergeometric function, Bessel functions, functions of the parabolic cylinder and of the paraboloid of revolution, the incomplete Gamma functions and related functions, orthogonal polynomials, spherical and hyperspherical harmonic polynomials and elliptic functions and integrals.—E.C.P.

Additions to the Library

Archer, H. and E. Pine. To Perish Never. Cassell & Co. Ltd., 1954.

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Gatland, K. W. Development of the Guided Missile. 2nd Ed. Iliffe, 1954.

Gumbel, E. J. Statistical Theory of Extreme Values and some Practical Applications. U.S.G.P.O., 1954.

Hirschfelder, J. O., C. F. Curtiss and R. Byron Bird. Molecular Theory of Gases and Liquids. John Wiley, 1954.

Lanchbery, Edward (*Editor*). Neville Duke's Book of Flying. Cassell & Co. Ltd., 1954.

Poole, J. H. J. The Evolution of the Earth's Atmosphere (Sc. Proc. R. Dublin Society XXV, 16). R. Dublin Society, 1951.

Rolls-Royce. Dart R. Da.6 Rating Engine. Estimated Performance Data and Installation Notes. Rolls-Royce, 1954.

Rosenhead, L. *et al.* A Selection of Graphs for Use in Calculations of Compressible Airflow. Clarendon Press, 1954.

Saunders, H. St. G. Royal Air Force 1939-45. Volume III—The Fight is Won. H.M.S.O., 1954.

Taylor, J. W. R. Civil Aircraft Markings. Ian Allan Ltd., 1954.

Vinogradov, I. M. (*translated by* K. F. Roth and A. Davenport). The Method of Trigonometrical Sums in the Theory of Numbers. Interscience Publishers Ltd., 1954.

Wax, Nelson (*Editor*). Selected Papers on Noise and Stochastic Processes. Dover Publications, 1954.

Whitehead, T. N. The Design and Use of Instruments and Accurate Mechanism; Underlying Principles. Dover Publications, 1954.

Reports

AERODYNAMICS

Boundary Layer

Turbulent boundary layer control by ramps or wedges. A. V. Stephens and G. A. Collins. A.R.L. Australia Report A.85 (April 1954).

Part I describes preliminary experiments of a new device for the delay or elimination of turbulent separation. The device is essentially a means of redistributing the total pressure in a boundary layer by means of wedge shaped excrescences on the surface. Part II deals with the continuation of these experiments. The operation of boundary layer wedges for controlling turbulent separation was investigated in an adverse pressure gradient on the upper surface of a wing. A new type of wedge in the form of a flat ramp was developed. These were applied to various wings and flaps with significant improvements in the lift characteristics.—(1.1.4.1).

Direct measurements of skin friction. Satish Dhawan. N.A.C.A. Report 1121 (1953).

A device was developed to measure local skin friction on a flat plate by measuring the force exerted on a very small movable part of the flat plate. The apparatus was applied to measurements in the low-speed range, for both laminar and turbulent boundary layers. Measurements were made in high-speed subsonic flow and turbulent-skin-friction coefficients were determined up to a Mach number of about 0·8. A few measurements were also made in supersonic flow.—(1.1.0).

Characteristics of turbulence in a boundary layer with zero pressure gradient. P. S. Klebanoff. N.A.C.A. Technical Note 3178 (*July* 1954).

The results of an experimental investigation of a turbulent boundary layer with zero pressure gradient are presented. Measurements with the hot-wire anemometer were made of turbulent energy and turbulent shear stress, probability density and flattening factor of u-fluctuation, spectra of turbulent energy and shear stress, and turbulent dissipation. —(1.1.3).

Heat, mass, and momentum transfer for flow over a flat plate with blowing or suction. H. S. Mickley, R. C. Ross, A. L. Squyers, and W. E. Stewart. N.A.C.A. Technical Note 3208 (*July* 1954).

The effect on the boundary layer of sucking or blowing air through a porous flat plate into or out of a main air stream flowing parallel to the plate was studied theoretically and experimentally. Laminar boundary layer theory was used to calculate transfer coefficients and velocity, temperature, and concentration profiles in laminar and zero Euler number flow under certain restricted mass transfer conditions. For turbulent flow film theory was used to predict the effect of mass transfer on friction, heat, and mass transfer coefficients. Experimental measurements of velocity and temperature profiles and of friction and heat transfer coefficients were made over a range of flow conditions.—(1.1.5 × 1.9.2).

Compressible Flow

The rapid, accurate prediction of pressure on non-lifting ogival heads of arbitrary shape, at supersonic speeds. W. Bolton-Shaw and H. K. Zienkiewicz. A.R.C. Current Paper No. 154 (1954).

Five methods are developed for determining the pressure distribution on an arbitrary, pointed, convex axi-symmetric head shape, of which the ordinate and its first three derivatives are everywhere continuous. When the geometric details are specified, the time required to pressure plot a head shape by these methods is from approximately 20 minutes to 3 hours. The methods were checked in five trial cases against accurate pressure distributions, obtained by using van Dyke's second order theory, and generally gave very good agreement. The best method to use of the five depends on head shape and on the speed and accuracy required, and recommended methods are given.—(1.2.3).

Shock-turbulence interaction and the generation of noise. H. S. Ribner. N.A.C.A. Technical Note 3255 (*July* 1954).

Inter-action of convected field of turbulence with shock wave is analysed to yield modified turbulence, entropy spottiness, and noise generated downstream of the shock. Analysis is generalisation of single-spectrum-wave treatment of T.N.2864. Formulae for spectra and correlations are obtained. Numerical calculations yield curves of r.m.s. velocity components, temperature pressure, and noise in db against Mach number for $M=1$ to ∞; both isotropic and strongly axi-symmetric (lateral/longitudinal = 36/1) initial turbulence are treated.—(1.2.3.2).

Control Surfaces

An investigation into the rolling power and aileron reversal characteristics of swept wings. A. V. Coles and R. J. Margetts. A.R.C. Current Paper No. 159 (1954).

Existing theories relating to swept wings are reviewed briefly and the results obtained from a programme of tests on flexible swept wings are presented. The wings tested were of constant span, aspect ratio and area and had angles of sweep ranging from 15° sweepforward to 60° sweepback. The tests were made in a low-speed wind-tunnel, compressibility effects being absent.—(1.3.1 × 2).

Note on the dynamic characteristics of servo-tab systems of control. D. Adamson and D. J. Lyons. R. & M. 2853 (*April* 1948, *published* 1954).

Generalised curves have been constructed from which estimates can be made of those dynamic characteristics of the servo-tab type of control which are of chief interest to the designer. The characteristics evaluated for two specific cases, a 50,000 lb. and a 300,000 lb. aircraft, indicate no special problems to the designer or pilot excepting overshoot of the control at low flying speeds. Elastic stops are considered to be the most promising solution to this.—(1.3.5).

Fluid Dynamics

A new law of similarity for profiles, valid in the transonic region. K. Oswatitsch. R. & M. No. 2715 (*June* 1947, *published* 1954).

A new law of similarity is given, valid for slender profiles in mixed transonic flow with negligible viscosity, according to which the cube of the Prandtl factor of any critical Mach number is proportional to the thickness ratio. It is shown that this rule, and that of von Kármán for flow at sonic speed, are valid for shock-waves within the range over which the shock loss is proportional to the cube of the pressure rise. Experimental pressure distributions plotted according to this rule show good agreement, except for the position of the shock-wave on the surface.—(1.4).

Treatment of the stagnation point in arithmetical methods. A. Thom. R. & M. 2807 (*May* 1951, *published* 1954).

In using any of the Relaxation Techniques near a stagnation point difficulties arise if the variable is $\log 1/q$. This tends to infinity and the difference equation no longer represents adequately the differential equation without special modification. Methods are given whereby larger squares can be used than had been previously practicable. As little variation as possible has been introduced into the procedure so that if an electronic calculator is used, the alterations to the circuits would be reduced to a minimum.—(1.4.1).

Impingement of water droplets on N.A.C.A. 65A004 airfoil at 8° angle of attack. R. J. Brun, Helen M. Gallagher, and Dorothea E. Vogt. N.A.C.A. Technical Note 3155 (*July* 1954).

The trajectories of droplets in the air flowing past an N.A.C.A. 65A004 aerofoil at an angle of attack of 8° were determined. The amount of water in droplet form impinging on the aerofoil, the area of droplet impingement, and the rate of droplet impingement per unit area on the aerofoil surface were calculated from the trajectories and presented to cover a large range of flight and atmospheric conditions.

NOTE:—*The figures in parentheses at the end of each Summary are for office use only.*

These impingement characteristics are compared briefly with those previously reported for the same aerofoil at an angle of attack of 4°.—(1.4).

Theoretical investigation at subsonic speeds of the flow ahead of a slender inclined parabolic-arc body of revolution and correlation with experimental data obtained at low speeds. W. Letko and E. C. B. Danforth. N.A.C.A. Technical Note 3205 (*July* 1954).

Approximate formulas, based on subsonic linearised theory, are derived for the static-pressure increment and flow inclination ahead of slender inclined parabolic-arc bodies of revolution at subsonic speeds. Charts are presented from which these quantities can be estimated and some comparisons with experimental data obtained for a Mach number of 0·21. Consideration is given to the induced effects of the wing and mounting boom on flow-inclination measurements.—(1.4.1).

INTERNAL FLOW

An experimental study of three-dimensional high-speed air conditions in a cascade of axial-flow compressor blades. K. W. Todd. R. & M. 2792. (*October* 1949, *published* 1954).

Detailed investigations have been made by optical and physical methods in a high-speed wind tunnel of the flow characteristics of two compressor blade cascades.—(1.5.2.3).

STABILITY AND CONTROL

See also WINGS AND AEROFOILS AND AEROELASTICITY

La stabilité transversale de vol et quelques recherches de l'O.N.E.R.A. Maurice Roy. O.N.E.R.O. Publication No. 69 (1954).—(1.8.1).

Theoretical investigation of longitudinal response characteristics of a swept-wing fighter airplane having a normal-acceleration control system and a comparison with other types of systems. F. H. Stokes and C. W. Mathews. N.A.C.A. Technical Note 3191 (*July* 1954).

An analysis is made of a normal-acceleration control system as applied to a swept-wing fighter aeroplane. The results show the response characteristics of the aeroplane-autopilot combination. The effects that changes in altitude and Mach number have on these response characteristics and the effects of changes in the parameter settings of the system are investigated. A comparison is also made of the normal-acceleration system with pitch-attitude, pitching-velocity, and angle-of-attack systems.—(1.8.2).

THERMO-AERODYNAMICS

See also BOUNDARY LAYER

Experimental investigation of temperature recovery factors on a 10° *cone at angle of attack at a Mach number of* 3·12. *J. R. Jack and B. Moskowitz. N.A.C.A. Technical Note* 3256 (*July* 1954).

At angles of attack from 0° to 10° over the Reynolds number range ($1{\cdot}5$ to 8×10^6), an increase in angle of attack increased the equilibrium surface temperatures in the laminar and turbulent flow regions. The equilibrium surface temperatures in regions of probable cross-flow separation were in the same range as those obtained for fully turbulent flow. For the windward surface, local recovery factors in the fully laminar and turbulent regions were not significantly affected by changes in angle of attack.—(1.9.1).

WINGS AND AEROFOILS

See also TESTING AND INSTRUMENTS

The drag increase at high subsonic speeds. K. Oswatitsch. R. & M. 2716 (*October* 1947, *published* 1954).

The drag increase beyond the critical Mach number is calculated by modifying the supersonic part of the Kármán-Tsien pressure distribution on a profile. This is possible when the supersonic regions are not too large. The formula giving the modified pressure distribution is derived very roughly. It may give only one of the main effects appearing when supersonic speeds occur in the flow, and may be changed and calculated more exactly later.—(1.10.2.1).

Methods of approaching an accurate three-dimensional potential solution for a wing. H. C. Garner. R. & M. 2721 (*October* 1948, *published* 1954).

There is a great need for more accurate data on the aerodynamic derivatives of swept-back wings in order to solve problems of stability, control and flutter. As one step in the search for these data the estimation of the three-dimensional potential solution is essential, and if it is to be of value the degree of accuracy of any approximation must be known beyond question. Some fundamental aspects of the vortex-sheet theory for determining the distribution of lift on a finite wing are discussed together with the accuracy and limitations of some existing approximate forms of the theory. With special reference to the labour of computation an iterative approach to an accurate solution is suggested and the general mathematical expression for the distribution of lift required to give an exact solution for a Vee wing is considered.—(1.10.1.2).

Velocity distribution on thin tapered wings with fore-and-aft symmetry and spanwise constant thickness ratio at zero incidence. S. Neumark and J. Collingbourne. R. & M. 2858 (*June* 1951, *published* 1954).

This report is a continuation of three earlier ones by the present authors, and contains a theoretical investigation of subsonic flow past thin tapered unswept wings (of full or cropped-rhombus plan form), at zero incidence. Only the case of spanwise constant thickness ratio is considered in this first attempt although alternative cases also merit attention. The first order method of linear perturbation based on continuous systems of sources and sinks is shown to be still applicable to tapered wings, although mathematical difficulties are greatly increased.—(1.10.1.2).

Aérodynamique instationnaire des profils minces déformables. B. Fraeys de Veubeke. Bulletin du Service Technique de L'Aéronautique No. 25 (1953).—(1.10.1.1).

Lift and pitching moment at supersonic speeds due to constant vertical acceleration for thin swept back tapered wings with streamwise tips. Supersonic leading and trailing edges. Isabella J. Cole and K. Margolis. N.A.C.A. Technical Note 3196 (*July* 1954).

On the basis of linearised time-dependent supersonic flow theory, the lift and pitching-moment derivatives due to constant vertical acceleration are evaluated for a series of thin swept-back tapered wings with streamwise tips. The analysis is valid, in general, at those speeds for which the wing leading and trailing edges are both supersonic.—(1.10.1.2 × 1.8.2).

An experimental study of the lift and pressure distribution on a double-wedge profile at Mach numbers near shock attachment. W. G. Vincenti, D. W. Dugan, and E. R. Phelps. N.A.C.A. Technical Note 3225 (*July* 1954).

An account is given of measurements at low supersonic speeds of the pressure distribution on a double-wedge profile. The results cover the Mach number range from 1·166 to 1·377, which brackets the value (1·221) for bow-wave attachment at zero angle of attack. The results are discussed and compared with the findings of the transonic small-disturbance theory.—(1.10.2.1).

TESTING AND INSTRUMENTS

The manufacture of aerofoil models by tangent plane milling. R. S. Marriner. A.R.C. Current Paper No. 166 (1954).—(1.12 × 1.10).

Schlieren methods for observing high-speed flows. D. W. Holder and R. J. North. A.R.C. Current Paper No. 167 (1954).

Use of schlieren and direct-shadow methods for the visualisation of high-speed flow is discussed with the emphasis on the use of the methods in wind tunnel experiments. The techniques for observing flows which are two-dimensional or which possess axial symmetry have reached an advanced stage of development, and are satisfactory for most investigations of this kind; recent progress is reviewed in Part I. In contrast, comparatively little work has been done on the development of techniques for visualising the flow round finite wings and wing-body combinations. Several methods which may be useful for the study of flows of this type are described briefly in Part II, but further work is necessary before the value of these methods can be assessed.—(1.12).

Note on an application of the tilting plate method of Mach number variation for wind tunnel tests at low supersonic speeds. J. Seddon and L. Haverty. A.R.C. Current Paper No. 168 (1954).

When a flat plate mounted in the working section of a supersonic wind tunnel is inclined at an angle to the stream, there exists a region above the surface of the plate in which the Mach number is constant and different from the main stream value. In limited circumstances this region may be used as the test section and it is possible, by varying the angle of the plate, to obtain a continuous variation of test Mach number with the one fixed tunnel nozzle. This method of Mach number control can be particularly useful for making wind tunnel tests near $M = 1{\cdot}0$.—(1.12.1.3).

A simple method of computing C_D from wake traverses at high-subsonic speeds. J. S. Thompson. R. & M. No. 2914 (*December* 1944, *published* 1954).

This note gives a convenient method of obtaining C_D from a pitot-static traverse in an aerofoil wake, using Jones' equation as modified by Lock, Hilton and Goldstein for compressible flow. Charts are provided from which the integrand C_D' can easily be obtained for any point in the traverse, but it is shown that in nearly all cases an accuracy of 1 per cent. in C_D can be obtained by applying an integrating factor to the area under the total-head loss curve.—(1.12).

The high-speed laboratory of the Aerodynamics Division, N.P.L. D. W. Holder. R. & M. 2560 (*December* 1946, *published* 1954). *A.R.C. Monograph.*

The first part of this report describes the high-speed tunnel installation in the Aerodynamics Division of the National Physical Laboratory including a series of experiments which were made to investigate the influence of the design on the efficiency of an induced flow tunnel, and finally a description of the new 18×14-in. high-speed tunnel. The second part describes some of the experimental techniques which have been used. Many of these are similar in principle to those of low-speed tunnel practice, but some of them (*e.g.* schlieren and shadowgraph techniques) are peculiar to compressible-flow experiments. The third part of the report reviews the experimental results obtained in the high-speed tunnels during and immediately before the 1939-45 War.—(1,12.1).

Transonic flow past swept and unswept wings between parallel walls. W. Jacobs. F.F.A. Report 55 (1954).

This investigation deals with the drag of straight and swept wings between parallel walls at high subsonic Mach numbers as well as at Mach numbers in the transonic and supersonic ranges. From pressure measurements at zero angle of attack, the drag of the wings is computed. The main object is to provide an estimation of the wall interferences and thus, of the magnitude of the effective sweep angle of wings between parallel walls in the transonic range.—(1.12.1.2).

AEROELASTICITY

See also AERODYNAMICS, CONTROL SURFACES

The technique of flutter calculations. H. Templeton. A.R.C. Current Paper No. 172 (1954).

This report describes the basic principles on which theoretical flutter analyses are made, and illustrates them by some simple applications. The techniques employed are typical of those in current use in this country. Three Appendices give the two-dimensional aerodynamic derivatives for a wing-aileron-tab system, computational details of typical forms of solution, and an illustration of the use of resonance test modes in flutter calculations.—(2).

The rolling power of an elastic swept wing. E. G. Broadbent. R. & M. 2857 (*June* 1950, *published* 1954).

An iterative method of solution is given for the problem of loss in rolling power due to wing deformation. The method is applicable at subsonic speeds, and compressibility effects are allowed for, provided the variation of the aerodynamic derivatives with Mach number is known. The numerical labour involved in the solution is not great and the accuracy is considerably greater than can be achieved by the semi-rigid method.—(2).

Charts and approximate formulas for the estimation of aeroelastic effects on the loading of swept and unswept wings. F. W. Diederich and K. A. Foss. N.A.C.A. Report 1140 (1953).

Charts are presented for the estimation of aeroelastic effects on spanwise lift distribution, lift-curve slope, aerodynamic centre, and damping in roll of swept and unswept wings at subsonic and supersonic speeds. Some design considerations brought out by the results of this paper are discussed. (This report was formerly T.N.2608.)—($2 \times 1.8.2.1$).

DESIGN AND CONSTRUCTION

Drop hammer tests with three oleo strut models and three different shock strut oils at low temperatures. Kranz. N.A.C.A. Technical Memorandum 1372 (*July* 1954).

Drop hammer tests with different shock strut models and shock strut oils were performed at temperatures ranging to $-40°C$. The various shock strut models do not differ essentially regarding their springing and damping properties at low temperatures; however, the influence of the different shock strut oils on the springing properties at low temperatures varies greatly.—(4.2.2.3.1).

AIRCRAFT OPERATION

Aircraft fire extinguishment Part IV. evaluation of a bromochloromethane fire-extinguishing system for the XB-45 airplane. C. A. Hughes and C. M. Middlesworth. C.A.A. Technical development Report No. 240 (*June* 1954).

A bromochloromethane fire-extinguishing system, designed by Walter Kidde & Company, Inc., in accordance with U.S.A.F. Specification MIL-E-5352, was fabricated and installed in an XB-45 aeroplane power plant. Fire-extinguishing tests were conducted under simulated flight-fire emergency conditions to check the validity of certain phases of the specification and of certain design practices. Quantity requirements for extinguishing fires in each nacelle compartment were determined for bromochloromethane and dibromodifluoromethane.—(5.3).

Considerations on a large hydraulic jet catapult. U. T. Joyner and W. B. Horne. N.A.C.A. Technical Note 3203 (*July* 1954).

A survey of various types of catapults, which has been made in connection with the problem of accelerating a large (100,000 lb.) car along a track to a speed of 150 miles per hour, is given. A hydraulic jet catapult is indicated as the best suited among these catapult types for the purpose intended, and various design problems of this type are treated. Equations are given for calculating the performance of the jet and of the test car, and consideration is given to the physical conditions affecting the jet flow. Design procedures are presented for the jet nozzle and for the bucket on the car which receives the jet and imparts thrust to the car. The expected propulsive efficiency of the jet catapult is given and the effect of a side wind on the jet trajectory is calculated.—(5.5).

AIRPORTS

Analytical and simulation studies of several radar-vectoring procedures in the Washington, D.C., terminal area. S. M. Berkowitz and Ruth R. Doering. C.A.A. Technical Development Report 222 (*April* 1954).

This report summarises the comparative evaluations of three promising traffic control configurations for the Washington National Airport terminal area, which is approximately 40 miles in diameter. This 40-mile diameter represents the expected surveillance-radar coverage. Each of the three configurations, identified as phases 2, 6 and 7, uses within this 40-mile area two essentially independent control sectors (East and West), twin stacks to feed the approach gate, and radar-vectoring techniques. The three phases differed only in layout of facilities, in twin stacks, in clearance limits, and in procedures.—(6.5).

Evaluation of a controllable-beam runway light. M. S. Gilbert and H. J. Cory Pearson. C.A.A. Technical Development Report No. 238 (*June* 1954).

This report discusses the evaluation of a new type of controllable-beam runway light. The light was designed to provide an approach to an ideal candlepower distribution. the values of the candlepower for various angles of view

being derived from a functional analysis of runway-light requirements. A sample of the new type of light was tested photometrically, and the results were compared with the ideal candlepower distribution determined by theoretical analysis.—(6.3).

FLIGHT TESTING

The measurement of position error at high speeds and altitude by means of a trailing static head. K. W. Smith. A.R.C. Current Paper No. 160 (*June* 1952, *published* 1954).

The static position error of a service wing-tip leading edge pressure head installation has been measured on a Meteor VII by means of a trailing static head, developed especially for use at high speeds. These tests cover an altitude range from zero to 38,000 ft., and include measurements in "g" turns. The maximum Mach number reached was 0·84.—(13.1).

FUELS AND LUBRICANTS

Infra-red spectra of 47 *dicyclic hydrocarbons. J. H. Lamneck, H. F. Hipsher and Virginia O. Fenn. N.A.C.A. Technical Note* 3154 (*June* 1954).

The infra-red spectra are presented for 47 dicyclic hydrocarbons consisting of some alkyldiphenylmethanes, alkyldicyclohexylmethanes, alkylnaphthalenes, alkyltetralins, 1,3-diphenyl-2-alkylpropanes and 1,3-dicyclohexyl-2-alkylpropanes. The physical properties of these highly purified compounds are included for reference purposes.—(14.3).

HYDRODYNAMICS

Unsteady cavitating flow past curved obstacles. L. C. Woods. A.R.C. Current Paper No. 149 (1954).

The plane incompressible flow past two symmetric curved obstacles, between which is a finite constant pressure cavity, is calculated for the case when the cavity length and pressure are functions of time.—(17.1).

A review of porpoising instability of seaplanes. A. G. Smith and H. G. White. R. & M. 2852 (*February* 1944, *published* 1954).

A review has been made of the evidence on take-off and landing porpoising instability of seaplanes. The basic types of porpoising and their occurrence have been examined; full-scale results have been correlated with model-scale and theoretical results.—(17.2).

Aerodynamic characteristics of a refined deep-step planing-tail flying-boat hull with various forebody and afterbody shapes. J. M. Riebe and R. L. Naeseth. N.A.C.A. Report 1144 (1953).

An investigation was made in the Langley 300-m.p.h. 7- by 10-foot tunnel to determine the aerodynamic characteristics of a refined deep-step planing-tail hull with various forebody and afterbody shapes and, for comparison, a streamline body simulating the fuselage of a modern transport aeroplane.—(17.2).

INSTRUMENTS AND EQUIPMENT

A procedure for the design of air-heated ice-prevention systems. C. B. Neel. N.A.C.A. Technical Note 3130 (*June* 1954).

The procedure to be followed in the design of aircraft ice-prevention equipment in which the components are protected by means of internally circulated heated air is outlined. In addition to presentation of the required heat-transfer and air-pressure-loss equations, a simple electrical analogue is described which was devised to facilitate the design of an air-heated system. An illustration is given of the application of the analogue to a design problem.—(18.4).

MATERIALS

The effect of temperature on the strength of wood, plywood and glued joints. P. H. Sulzberger. A.R.C.C. Australia Report ACA-46 (*December* 1953).

Results show that all properties investigated are affected by temperature at some moisture content, in many instances to such an extent that serious error is introduced in mechanical testing and in design where this factor is not taken into account. Temperature coefficients may exceed one per cent. of the value at 20°C. per degree centigrade. Relations are given from which the effect of temperature on certain properties of a species can be estimated quantitatively with considerable confidence for a wide range of temperatures and moisture contents.—(21.3.5).

Filiform corrosion products on iron immersed in brine. P. F. Thompson and K. F. Lorking. A.R.L. Australia, Metallurgy Note 1 (*May* 1954).

A new type of filiform corrosion pustule has been observed on steel test specimens immersed in a commercial refrigerating brine containing chromate as a corrosion inhibitor. These filaments were secondary corrosion products growing from primary hemispherical pustules on the metal surface. The mechanism of the growth of the filaments is discussed.—(21).

The distribution of dislocations in linear arrays. A. K. Head and N. Louat. A.R.L. Australia Report Met. 2 (*May* 1954).

The anlysis is based on the assumption that an array of discrete dislocations may be replaced by a continuous distribution of smeared dislocation. The solutions of a number of problems of physical interest are investigated, including some in which dislocations of opposite sign are involved.—(21).

Survey of portions of the chromium-cobalt-nickel-molybdenum quaternary system at 1,200°*C. S. P. Rideout and P. A. Beck. N.A.C.A. Report* 1122 (1953).

A survey was made of portions of the chromium-cobalt-nickel-molybdenum quaternary system at 1,200°C. by means of microscopic and X-ray diffraction studies. The survey of these systems was confined to the determination of the boundaries of the face-centred cubic (alpha) solid solutions and of the phases co-existing with alpha at 1,200°C.—(21).

MATHEMATICS

*Le vrai problème du dépouillement des expériences. Pierre Vernotte. Publications Scientifiques et Techniques du Ministère de l'Air, France, No. N.T.*49.—(22).

MECHANICAL ENGINEERING

Fundamental study of erosion caused by steep pressure waves. B. G. Rightmire and J. M. Bonneville. N.A.C.A. Technical Note 3214 (*June* 1954).

A fundamental study of erosion caused by steep pressure waves has been carried out. It is believed that the study gives an insight to the possible causes of damage in high-speed sleeve bearings. In particular, the effect on annealed copper surfaces of steep-fronted pressure waves in oil has been studied, the general conclusion being that cavitation of the oil is the probable cause of damage.—(23.2.1).

NAVIGATION

The type III portable pictorial computer. Part II. Technical and operational evaluation. E. M. Blount Hugh, A. Kay, R. E. McCormick, F. S. McKnight, M. H. Yost. C.A.A. Technical Development Report No. 209 (*June* 1954).

By displaying position information directly on a chart, the Type III unit reduces the mental workload required of the pilot for navigation. This is particularly advantageous for flight in involved terminal-area patterns. Fewer "lost" aircraft incidents should occur for aircraft equipped with this type of pictorial display.—(26).

A dual-control course-line computer CAA type 1*A. W. L. Seibert, A. L. Saunders and L. E. Setzer. C.A.A. Technical Development Report No.* 244 (*July* 1954).

The CAA Type 1A course-line computer features dual control units, a factor which enables the pilot to preset the parameters of the destination or of the way point on one of the control heads without interrupting the operation of the computer. The pilot can then alternate control heads in flight. Thus, when the flight plan is comprised of a succession of waypoints, the guidance which the course-line computer furnishes becomes more convenient to use than that guidance furnished when a single control head is employed.—(26).

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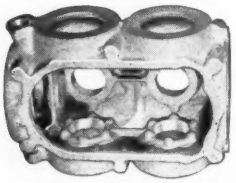

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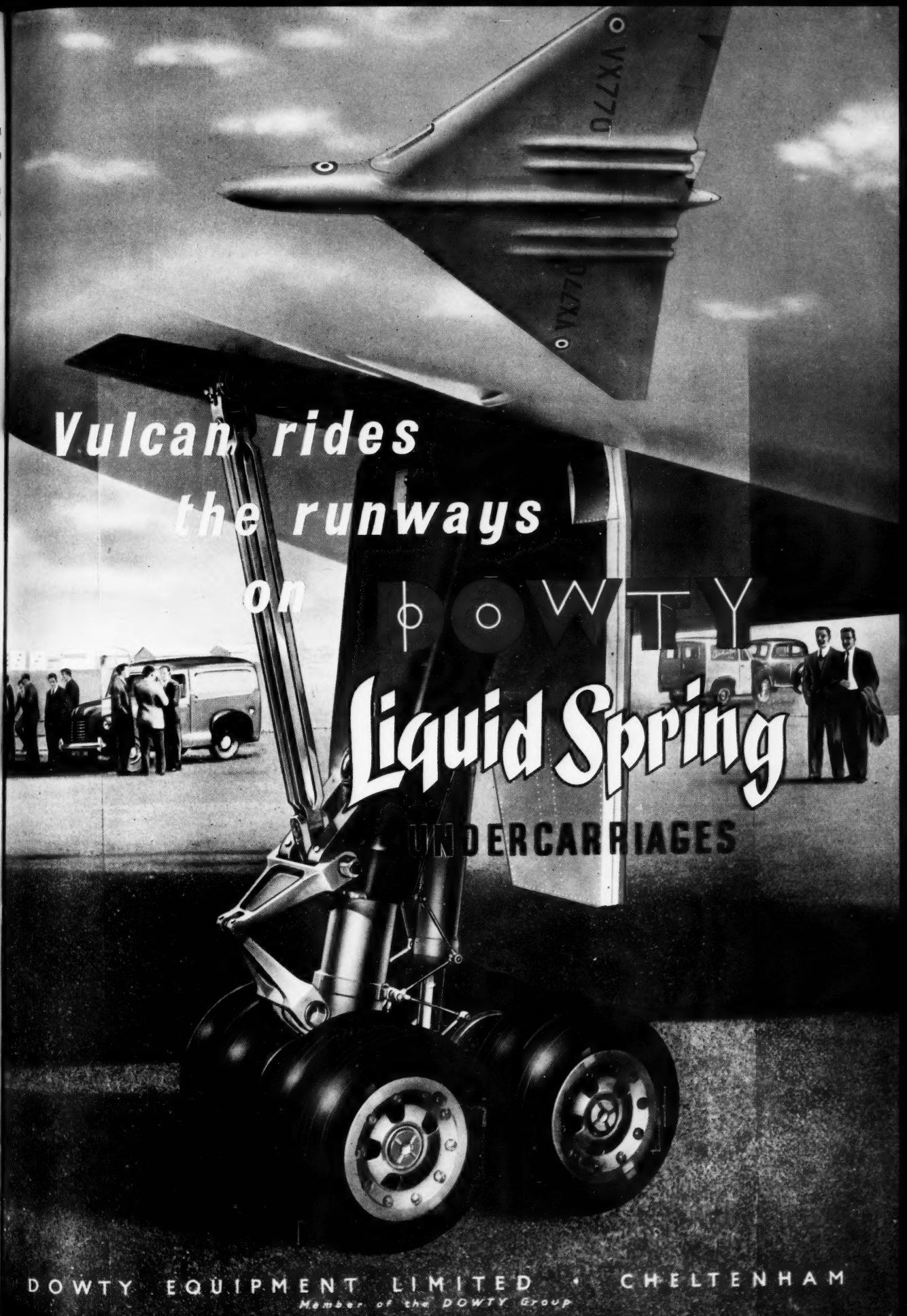
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